Survey of Primitive Idempotents in Cyclic Group Algebra

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Abstract : This paper gives a brief survey of primitive idempotents in cyclic group algebras for different cases. The expressions for these idempotents are listed. Initially the structure for cyclic codes is given.
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I Introduction

The theory of detecting and correcting the error was first introduced by Claude Shannon in 1948 in his paper "Mathematical Theory of Communication". In his paper Shannon said that we can easily transmit any information by coding. There are number of special codes such as cyclic codes, Linear codes, Group codes, polynomial codes etc. Our interest in this paper is to study a very important class of codes called "Cyclic Codes".

In general while examine cyclic codes over finite field F most often the code words are presented in polynomial form. The correspondence between the n- vector C $=c_0c_1\ldots c_{n-1}$ over F and the polynomial $c(x)= c_0 + c_1x+\ldots+c_{n-1}x^{n-1}$ in F[x] of at most n-1 degree is one to one and onto. This allows us the latitude of the vector notation C and the polynomial notation c(x) inter changeably. Notice that if $c(x) = c_0 + c_1x+\ldots+c_{n-1}x^{n-1}$ then $xc(x) = c_{n-1}x^n + c_0 x+c_1x^2+\ldots+c_{n-2}x^{n-1}$ represents the code word C cyclically shifted one to the right if x^n were set equal to 1. Equivalently, as the cyclic code *C* is invariant under a cyclic shift implies that if c(x) is in *C* then so is xc(x) provided we multiply modulo x^n-1. This fact allows us for studying cyclic codes in the residue class ring

$$R_n = \frac{F(x)}{< x^n - 1 >}.$$

It is also easily seen that

$$R_n = \frac{F(x)}{< x^n - 1 >} \cong FC_n$$

where FC_n is the group algebra of the cyclic group C_n of order n over the field F. Under the correspondence of the vectors with polynomials as given above, cyclic codes are ideals in R_n and ideals in R_n are cyclic codes. Therefore, the study of cyclic code over the finite field F is equivalent to the study of the ideals in R_n or FC_n, the group algebra of the cyclic group C_n of order n over the field F. It is well known that the study of ideals in R_n completely depend on factorization of x^n-1 over F. Interesting it is also well known fact x^n-1 has no repeated irreducible factors if and only if g.c.d(n, char(F)) =1. As F[x] is principal ideal domain then so is R_n. Thus a cyclic code, being ideal in R_n, may have a variety of generating polynomial.

Through out for our discussion of cyclic codes we make the basic assumption that char(F)- the characteristic of the field F does not divide n- the length of the cyclic codes. This assumption also implies that R_n is semi-simple and thus the Weddernburn structure theorem is applicable. The theory of cyclic codes with g.c.d(n, char(F)) ≠1 is discussed in [van,cag1991], but today these "repeated roots" cyclic codes don't seems to be of much interest.

Besides the generating polynomial, there are many other polynomials that can be used to generate a cyclic code. One such polynomial called an idempotent generator, can also be used to generate a cyclic code. As the ring R_n is semi-simple therefore each ideal in R_n contains a unique idempotent which also generates the ideal. This idempotent is called the generating idempotent of the corresponding cyclic code. The idempotent generating the minimal ideal (minimal code) in R_n is called a **Primitive idempotent.**

It is well known that the generating polynomial g(x) of the ideal in R_n is a factor of x^n-1. Thus the study of ideal through the generating polynomial depends on the factorization of x^n-1 over the field F. But the factorization of x^n-1 into its irreducible factors in itself is a very difficult problem. To overcome the problem of factorization, we deal with the idempotents that generates the ideals. These idempotents then help us to describe the cyclic codes completely.

Let F=GF(l) be a finite field of order l and n be any integer such that char (F) does not divide n.

Consider the set

$$S = \{0, 1, 2,\ldots,n-1\}.$$

For a, b $\in$ S, say that a ~b if $a \equiv b l^i$ (mod n) for some integer $i \geq 0$. This is an equivalence relation on set S. The equivalence classes of this relation are called l - cyclotomic class modulo n. The l -cyclotomic coset modulo n containing $s \in S$ is

$$C_s = \{s, sl, sl^2, \ldots, sl^{t_s-1}\},$$

where t_s is the least positive integer with $sl^{t_s-1} \equiv s$ (mod n). Each cyclotomic coset is associated with an irreducible polynomial in the semi simple ring $R_n = \frac{F[x]}{<x^n-1>}$ and hence is also associated with a primitive idempotent in R_n that generates a minimal ideal in R_n equivalently a minimal cyclic code over F. The number of l - cyclotomic class modulo n depends on t, the multiplicative order of l modulo n, where $1 \leq t \leq \varphi(n)$. Throughout the whole discussion we will assume that F is the field of order q, the group is cyclic and is generated by g.

Primitive Idempotents in Cyclic Group Algebras:

Let $C_{p^n} = <g>$ be the Cyclic Group. Berman[2] described an explicit expressions for the (n+1) primitive idempotents in FC_{p^n} (without proof), where q is the order of the field, is a prime number such that (q,p)=1 and is primitive root modulo p^i for all $i \geq 1$. Blake and Mullin[3] declared that it is tedious to verify that these expressions are idempotents in FC_{p^n}. Arora and Pruthi[1] gave an explicit expressions for the (n+1) primitive idempotents in FG(the group algebra of the cyclic group G of order p^n,p is an odd prime,n>1) over the finite field F of prime power order q with (q,p) = 1 and q is primitive root modulo p^n.

In 1997, Arora-Pruthi[1] descried the (n+1) primitive idempotents of FC_{p^n} given by:

$$e_0 = \frac{1}{p^n}\left(1 + \sum_{i=1}^{n} \bar{C}_i\right)$$

and for $1 \leq i \leq n$,

$$e_i = \frac{1}{p^n}\left((p-1)\left(1 + \bar{C}_{i+1} + \bar{C}_{i+2} + \ldots + \bar{C}_n\right) - \bar{C}_i\right)$$

where

$$\bar{C}_i = \sum_{s \in C_i} g^s$$

In 1999, Pruthi-Arora[24] described the (2n+2) primitive idempotents in FC_{2p^n} given by:

$$e_0 = \frac{1}{2p^n}\left(\sum_{j=1}^{n+1} \bar{C}_j^* + \sum_{j=1}^{n+1} \bar{C}_j\right)$$

$$\eta_0 = \frac{1}{2p^n}\left(\sum_{j=1}^{n+1} \bar{C}_j^* - \sum_{j=1}^{n+1} \bar{C}_j\right)$$

and for $1 \leq i \leq n$,

$$e_i = \frac{1}{2p^{n-i+1}}\left((p-1)\left(\sum_{j=i+1}^{n+1}\left(\bar{C}_j^* + \bar{C}_j\right) - \left(\bar{C}_i^* + \bar{C}_i\right)\right)\right)$$

$$\eta_i = \frac{1}{2p^{n-i+1}}\left((p-1)\left(\sum_{j=i+1}^{n+1}\left(\bar{C}_j^* - \bar{C}_j\right) - \left(\bar{C}_i^* - \bar{C}_i\right)\right)\right)$$

where

$$\bar{C}_i = \sum_{s \in C_i} g^s$$

and

$$\bar{C}_i^* = \sum_{s \in C_i^*} g^s$$

In 2001, Manju Pruthi[4] described (m+1) primitive idempotents in $F[x]/<x^{2^m}-1>$ given by:

$$e_0 = \frac{1}{2^m}\left(1+\sum_{i=1}^{n}\bar{C}_i\right)$$

And for $1 \le i \le m$,

$$e_i = \frac{1}{2^{m-i+1}}\left(\left(1+\bar{C}_{i+1}+\bar{C}_{i+2}+\ldots+\bar{C}_m\right)-\bar{C}_i\right)$$

where

$$\bar{C}_i = \sum_{s\in C_i} g^s$$

In 2002, Arora, Batra, Cohen and Pruthi[5] described (2n+1) primitive idempotents given by:

$$e_0 = \frac{1}{p^n}\left(1+\sum_{j=1}^{n}\bar{C}_j\right)$$

and

$$e_i = \frac{1}{2}\left(Y_i+\theta G_i\right)$$

$$e_i^* = \frac{1}{2}\left(Y_i-\theta G_i\right)$$

where $Y_i = \frac{1}{2p^{n-i+1}}\left(\left(p-1\right)\left(1+\bar{C}_{i+1}+\bar{C}_{i+2}+\ldots+\bar{C}_n\right)-\bar{C}_i\right)$

and $G_i = \frac{1}{p^{n-i+1}}\left(\bar{C}_i-\bar{C}_i^*\right)$

for $1 \le i \le n$, where if $\theta^2 = p$ if p = 4k+1 and $\theta^2 = -p$ if p = 4k-1

where $\bar{C}_i = \sum_{s\in C_i} g^s$

and $\bar{C}_i^* = \sum_{s\in C_i^*} g^s$

Bakshi and Raka[6] gave the explicit expressions for the 3n+2 primitive idempotents in $FC_{p^n r}$, where p,q,r are distinct odd primes, and q is a primitive root modulo p^n and is a primitive root modulo r. And $\left(\frac{\phi(p^n)}{2},\frac{\phi(r)}{2}\right)=1$.

The primitive idempotents are given by:

$$e_0 = \frac{1}{p^n q}\left(\bar{C}_0+\bar{C}_{p^n}+\sum_{j=0}^{n-1}\left(\bar{C}_{p^j r}+\bar{C}_{p^j}+\bar{C}_{ap^j}\right)\right),$$

and $$e_{p^n} = \frac{1}{p^n q}\left\{(q-1)-\sum_{j=0}^{n}\bar{C}_{p^j}-\sum_{j=0}^{n-1}\bar{C}_{ap^j}+(q-1)\sum_{j=0}^{n-1}\bar{C}_{p^j r}\right\}$$

for $0 \le j \le n-1$

$$e_{p^j r} = \frac{p-1}{p^{j+1}r}\left\{1+\sum_{i=n-j}^{n-1}\left(\bar{C}_{p^i}+\bar{C}_{ap^i}+\bar{C}_{p^i r}+\bar{C}_{p^n}\right)\right\}-\frac{1}{p^{j+1}r}\left(\bar{C}_{p^{n-j-1}}+\bar{C}_{ap^{n-j-1}}+\bar{C}_{p^{n-j-1}r}\right)$$

For $0 \le j \le n-1$, remaining 2n primitive idempotents are :

$$e_j = \frac{(p-1)(r-1)}{2p^{j+1}r}+\frac{1}{p^{n+j}r}A_{n-1}\bar{C}_{p^{n-j-1}}+\frac{1}{p^{n+j}r}B_{n-1}\bar{C}_{ap^{n-j-1}}$$
$$-\frac{p-1}{2p^{j+1}r}\left\{\sum_{i=n-j}^{n}\bar{C}_{p^i}+\sum_{i=n-j}^{n-1}\bar{C}_{ap^i}\right\}-\frac{r-1}{2p^{j+1}r}\bar{C}_{p^{n-j-1}r}+\frac{(p-1)(r-1)}{2p^{j+1}r}\left\{\sum_{i=n-j}^{n-1}\bar{C}_{p^i r}\right\}$$

$$e_j^* = \frac{(p-1)(r-1)}{2p^{j+1}r}+\frac{1}{p^{n+j}r}B_{n-1}\bar{C}_{p^{n-j-1}}+\frac{1}{p^{n+j}r}A_{n-1}\bar{C}_{ap^{n-j-1}}$$
$$-\frac{p-1}{2p^{j+1}r}\left\{\sum_{i=n-j}^{n}\bar{C}_{p^i}+\sum_{i=n-j}^{n-1}\bar{C}_{ap^i}\right\}-\frac{r-1}{2p^{j+1}r}\bar{C}_{p^{n-j-1}r}+\frac{(p-1)(r-1)}{2p^{j+1}r}\left\{\sum_{i=n-j}^{n-1}\bar{C}_{p^i r}\right\}$$

where $\bar{C}_i = \sum_{s\in C_i} g^s$ and

$$A_{n-1} = \begin{cases} \frac{p^{n-1}(1+\alpha)}{2}, (\alpha^2 = pr) if q^{\frac{\phi(p^n r)}{2}} \equiv -1(\bmod p^n q) \\ \frac{p^{n-1}(1+\beta)}{2}, (\beta^2 = -pr) if q^{\frac{\phi(p^n r)}{2}} \not\equiv -1(\bmod p^n q) \end{cases}$$

$$B_{n-1} = \begin{cases} \frac{p^{n-1}(1-\alpha)}{2}, (\alpha^2 = pr) if q^{\frac{\phi(p^n r)}{2}} \equiv -1(\bmod p^n q) \\ \frac{p^{n-1}(1-\beta)}{2}, (\beta^2 = -pr) if q^{\frac{\phi(p^n r)}{2}} \not\equiv -1(\bmod p^n q) \end{cases}$$

Sharma, Bakshi, Dumir and Raka[7] described the primitive idempotents in FC_{p^n} where q is an odd prime power, may not be a primitive root mod p^n, p is an odd prime with (q,p) = 1 and order of q modulo p is f, $\left(\frac{p-1}{f}, q\right) = 1$ and $q^f = 1 + p\lambda$. Also p does not divide λ ($n \geq 2$) and (e,q) = 1, where $p = 1 + ef$.

If q is primitive root modulo p then $f = p - 1$.

The (en+1) primitive idempotents in FC_{p^n} are given by

$$e_0 = \frac{1}{p^n}\left(1 + g + g^2 + \ldots + g^{p^n - 1}\right),$$

$$e_{p^j} = \frac{f}{p^{j+1}} \sum_{\substack{i=0 \\ p^{n-j}|i}}^{p^n-1} g^i + \frac{1}{p^{j+1}}\left\{\rho_0 \sum_{i \in C_{p^{n-j-1}}} g^j + \rho_1 \sum_{i \in C_{p^{n-j-1}}} g^{aj} + \rho_2 \sum_{i \in C_{p^{n-j-1}}} g^{a^2 j} + \ldots + \rho_{e-1} \sum_{i \in C_{p^{n-j-1}}} g^{a^{e-1} j}\right\},$$

$$e_{ap^j} = \frac{f}{p^{j+1}} \sum_{\substack{i=0 \\ p^{n-j}|i}}^{p^n-1} g^i + \frac{1}{p^{j+1}}\left\{\rho_1 \sum_{i \in C_{p^{n-j-1}}} g^j + \rho_2 \sum_{i \in C_{p^{n-j-1}}} g^{aj} + \rho_3 \sum_{i \in C_{p^{n-j-1}}} g^{a^2 j} + \ldots + \rho_0 \sum_{i \in C_{p^{n-j-1}}} g^{a^{e-1} j}\right\},$$

...

$$e_{a^{e-1}p^j} = \frac{f}{p^{j+1}} \sum_{\substack{i=0 \\ p^{n-j}|i}}^{p^n-1} g^i + \frac{1}{p^{j+1}}\left\{\rho_{e-1} \sum_{i \in C_{p^{n-j-1}}} g^j + \rho_0 \sum_{i \in C_{p^{n-j-1}}} g^{aj} + \rho_2 \sum_{i \in C_{p^{n-j-1}}} g^{a^2 j} + \ldots + \rho_{e-2} \sum_{i \in C_{p^{n-j-1}}} g^{a^{e-1} j}\right\}$$

where ρ_0 is an eigenvalue of the matrix Δ and ($\rho_0, \rho_1, \rho_2, \ldots, \rho_{e-1}$) is the eigen vector corresponding to ρ_0 . The matrix Δ is given by

$$\Delta = \begin{pmatrix} A_{00} - f & A_{01} - f & A_{02} - f & \ldots & A_{0(e-1)} - f \\ A_{10} & A_{11} & A_{12} & \ldots & A_{1(e-1)} \\ A_{20} & A_{21} & A_{22} & \ldots & A_{2(e-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{(e-1)0} & A_{(e-1)1} & A_{(e-1)2} & \cdots & A_{(e-1)(e-1)} \end{pmatrix}$$

if f is even, and is given by

$$\Delta = \begin{pmatrix} A_{00} & A_{01} & A_{02} & \ldots & A_{0(e-1)} \\ A_{10} & A_{11} & A_{12} & \ldots & A_{1(e-1)} \\ A_{20} & A_{21} & A_{22} & \ldots & A_{2(e-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{\left(\frac{e}{2}\right)0} - f & A_{\left(\frac{e}{2}\right)1} - f & A_{\left(\frac{e}{2}\right)2} - f & \ldots & A_{\left(\frac{e}{2}\right)(e-1)} - f \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{(e-1)0} & A_{(e-1)1} & A_{(e-1)2} & \cdots & A_{(e-1)(e-1)} \end{pmatrix}$$

if f is odd, where A_{ij} is the number of ordered pairs (s,t), such that

$g^{es+i} + 1 = g^{et+j}$, $0 \leq s,t \leq f - 1$.

Arora, Batra and Cohen[8] discussed about the complete set of primitive idempotents in FC_{2^n} , the semi-simple group algebra of the cyclic group C_{2^n} of order 2^n($n \geq 2$) Also q is an odd prime with some prime power, where q has order $\frac{\phi(2^n)}{2}$ modulo 2^n.

The 2n-1 primitive idempotents for q = 8k+3 are given by

$e_0 = Y_0$,

$e_{n-1} = Y_2$,

$e_n = Y_1$

and for $1 \leq i \leq n-2$,

$$e_i = \frac{1}{2}\left(Y_{i+2} + 2\theta G_i\right),$$

$$e_i^* = \frac{1}{2}\left(Y_{i+2} - 2\theta G_i\right),$$

where $\theta := \sqrt{-2} \in GF(l) \subseteq F$

The 2n primitive idempotents for q = 8k-3 are given by

$$e_0 = Y_0,$$

$$e_n = Y_1$$

and for $1 \le i \le n-1$,

$$e_i = \frac{1}{2}\left(Y_{i+1} + 2\theta G_i\right),$$

$$e_i^* = \frac{1}{2}\left(Y_{i+1} - 2\theta G_i\right),$$

where $\theta := \sqrt{-1} \in GF(l) \subseteq F$, and l = char. F

$$\bar{C}_i = \sum_{s \in C_i} g^s$$

$$\bar{C}_i^* = \sum_{s \in C_i^*} g^s$$

$$Y_i = \frac{1}{2^{n-i+1}}\left[\left(1 + \bar{C}_{i+1} + \bar{C}_{i+1}^* + \ldots + \bar{C}_{i+1} - (\bar{C}_i + \bar{C}_i^*)\right)\right]$$

for $1 \le i \le n$.

For $1 \le i \le n-2$ and q = 8k+3, $G_i = \frac{1}{2^{n-i+1}}(\bar{C}_i - \bar{C}_i^*)$.

For $1 \le i \le n-1$ and q = 8k-3, $G_i = \frac{1}{2^{n-i+1}}(\bar{C}_i - \bar{C}_i^*)$.

Arora and Batra[9] described the minimal quadratic residue cyclic codes of length 2^n.

If q is of the form 8k+3, then $\left\{q^i \mid 0 \le i \le 2^{n-2} - 1\right\}$, the set of integers modulo 2^n accounts for all the odd numbers of the form 8m+3 or 8m+1 .

The 2n-1 primitive idempotents for the case q = 8k+3 are given by

$$e_0 = \frac{1}{2^n}\left[1 + \sum_{i=1}^{n-2}\left(\bar{C}_i + \bar{C}_i^*\right) + \left(\bar{C}_0 + \bar{C}_{n-1} + \bar{C}_n\right)\right],$$

$$e_{n-1} = \frac{1}{2^{n-1}}\left[2\left(1 + \bar{C}_3 + \bar{C}_3^* + \ldots + \bar{C}_{n-1} + \bar{C}_n\right) - \left(\bar{C}_2 + \bar{C}_2^*\right)\right],$$

$$e_n = \frac{1}{2^n}\left[2\left(1 + \bar{C}_2 + \bar{C}_2^* + \ldots + \bar{C}_{n-1} + \bar{C}_n\right) - \left(\bar{C}_1 + \bar{C}_1^*\right)\right]$$

and for $1 \le i \le n-2$,

$$e_i = \frac{1}{2^{n-i+1}}\left[2\left\{\left(1 + \bar{C}_{i+3} + \bar{C}_{i+3}^* + \ldots + \bar{C}_{n-1} + \bar{C}_n\right) - \left(\bar{C}_{i+2} + \bar{C}_{i+2}^*\right)\right\} - \theta\left(\bar{C}_i - \bar{C}_i^*\right)\right],$$

$$e_i^* = \frac{1}{2^{n-i+1}}\left[2\left\{\left(1 + \bar{C}_{i+3} + \bar{C}_{i+3}^* + \ldots + \bar{C}_{n-1} + \bar{C}_n\right) - \left(\bar{C}_{i+2} + \bar{C}_{i+2}^*\right)\right\} + \theta\left(\bar{C}_i - \bar{C}_i^*\right)\right],$$

where $\theta := \sqrt{-2} \in GF(l) \subseteq F$ and l = char. F

The 2n primitive idempotents for the case q = 8k-3 are given by

$$e_0 = \frac{1}{2^n}\left[1 + \sum_{i=1}^{n-1}\left(\bar{C}_i + \bar{C}_i^*\right) + \bar{C}_n\right],$$

$$e_n = \frac{1}{2^n}\left[\left(1 + \bar{C}_2 + \bar{C}_2^* + \ldots + \bar{C}_{n-1} + \bar{C}_{n-1}^* + \bar{C}_n\right) - \left(\bar{C}_1 + \bar{C}_1^*\right)\right]$$

and for $1 \le i \le n-1$,

$$e_i = \frac{1}{2^{n-i+1}}\left[\left\{\left(1 + \bar{C}_{i+2} + \bar{C}_{i+2}^* + \ldots + \bar{C}_n\right) - \left(\bar{C}_{i+1} + \bar{C}_{i+1}^*\right)\right\} - \theta\left(\bar{C}_i - \bar{C}_i^*\right)\right],$$

$$e_i^* = \frac{1}{2^{n-i+1}}\left[\left\{\left(1 + \bar{C}_{i+2} + \bar{C}_{i+2}^* + \ldots + \bar{C}_n\right) - \left(\bar{C}_{i+1} + \bar{C}_{i+1}^*\right)\right\} + \theta\left(\bar{C}_i - \bar{C}_i^*\right)\right],$$

where $\theta := \sqrt{-1} \in GF(l) \subseteq F$, and

$$\bar{C}_i = \sum_{s \in C_i} g^s$$

$$\bar{C}_i^* = \sum_{s \in C_i^*} g^s$$

for $1 \le i \le n$ .Arora and Batra described the primitive idempotents given by:

$e_0 = \overline{E_0}$

$$\eta_0 = \overline{E_0^*}$$

for $1 \le i \le n$

$$e_i = \frac{1}{2}\left(\overline{E}_i + \theta \overline{G}_i\right)$$

$$e_i^* = \frac{1}{2}\left(\overline{E}_i - \theta \overline{G}_i\right)$$

$$\eta_i = \frac{1}{2}\left(\overline{E}_i^* + \theta \overline{G}_i^*\right)$$

$$\eta_i^* = \frac{1}{2}\left(\overline{E_i^*} - \theta \overline{G}_i^*\right)$$

where

$\theta^2 = p$ if $p \equiv 1(\text{mod } 4)$, and

$\theta^2 = -p$, if $p \equiv -1(\text{mod } 4)$

and

$$\overline{E}_i = \frac{1}{2p^{n-i+1}}\left[(p-1)\left\{\left(\overline{C}_{p^i} + \overline{C}_{hp^i}\right) + \left(\overline{C}_{2p^i} + \overline{C}_{2hp^i}\right) + \ldots + \left(1 + \overline{C}_{p^n}\right)\right\} - \left\{\left(\overline{C}_{p^{i-1}} + \overline{C}_{hp^{i-1}}\right) + \left(\overline{C}_{2p^{i-1}} + \overline{C}_{2hp^{i-1}}\right)\right\}\right]$$

$$\overline{E}_i^* = \frac{1}{2p^{n-i+1}}\left[(p-1)\left\{\left(\overline{C}_{2p^i} + \overline{C}_{2hp^i}\right) - \left(\overline{C}_{p^i} + \overline{C}_{hp^i}\right) + \ldots + \left(1 + \overline{C}_{p^n}\right)\right\} - \left\{\left(\overline{C}_{2p^{i-1}} + \overline{C}_{2hp^{i-1}}\right) - \left(\overline{C}_{p^{i-1}} + \overline{C}_{hp^{i-1}}\right)\right\}\right]$$

If 2 is quadratic residue modulo p, then

$$\overline{G}_i = \frac{1}{2p^{n-i+1}}\left[\left(\overline{C}_{p^{i-1}} + \overline{C}_{2p^{i-1}}\right) - \left(\overline{C}_{hp^{i-1}} + \overline{C}_{2hp^{i-1}}\right)\right]$$

$$\overline{G}_i^* = \frac{1}{2p^{n-i+1}}\left[\left(\overline{C}_{2p^{i-1}} - \overline{C}_{p^{i-1}}\right) - \left(\overline{C}_{2hp^{i-1}} - \overline{C}_{hp^{i-1}}\right)\right]$$

If 2 is quadratic non-residue modulo p, then

$$\overline{G}_i = \frac{1}{2p^{n-i+1}}\left[\left(\overline{C}_{p^{i-1}} + \overline{C}_{2hp^{i-1}}\right) - \left(\overline{C}_{hp^{i-1}} + \overline{C}_{2p^{i-1}}\right)\right]$$

$$\overline{G}_i^* = \frac{1}{2p^{n-i+1}}\left[\left(\overline{C}_{2hp^{i-1}} - \overline{C}_{p^{i-1}}\right) - \left(\overline{C}_{2p^{i-1}} - \overline{C}_{hp^{i-1}}\right)\right]$$

$$for 1 \le i \le n$$

$$\overline{C}_{p^{i-1}} = \sum_{s \in C_{p^{i-1}}} g^s$$

$$\overline{C}_{hp^{i-1}} = \sum_{s \in C_{hp^{i-1}}} g^s$$

$$\overline{C}_{2p^{i-1}} = \sum_{s \in C_{2p^{i-1}}} g^s$$

$$\overline{C}_{2hp^{i-1}} = \sum_{s \in C_{2hp^{i-1}}} g^s$$

$$\overline{C}_0 = 1$$

$$\overline{C}_{p^n} = g^{p^n}$$

In 2010 S.K.Arora and Kulvir Singh [10] described an explicit expression for 4(n-1) primitive idempotents in FG , the semisimple group algebra of the cyclic group G of order 2^n ($n \ge 3$) over the finite field F of prime power order q, where q is quadratic residue modulo 2^n.

Then the primitive idempotents FC_{2^n} are given by

$$e_0 = Y_0\ ,$$

$$e_{(1),n} = Y_1$$

$$e_{(1),n-1} = \frac{1}{2}\left[Y_2 - 2\theta^2(G_{(1,2),1} + G_{(3,4),1})\right],$$

$$e_{(2),n-1} = \frac{1}{2}\left[Y_2 + 2\theta^2(G_{(1,2),1} + G_{(3,4),1})\right],$$

for $1 \le i \le n-2$

$$e_{(1),i}=\frac{1}{4}\left[Y_{i+2}-2\theta^2(G_{(1,2),i+1}+G_{(3,4),i+1})-4\theta(G_{(2,4),i}+\theta^2G_{(1,3),i})\right],$$

$$e_{(2),i}=\frac{1}{4}\left[Y_{i+2}+2\theta^2(G_{(1,2),i+1}+G_{(3,4),i+1})-4\theta(\theta^2G_{(2,4),i}+G_{(1,3),i})\right],$$

$$e_{(3),i}=\frac{1}{4}\left[Y_{i+2}-2\theta^2(G_{(1,2),i+1}+G_{(3,4),i+1})+4\theta(G_{(2,4),i}+\theta^2G_{(1,3),i})\right],$$

$$e_{(4),i}=\frac{1}{4}\left[Y_{i+2}+2\theta^2(G_{(1,2),i+1}+G_{(3,4),i+1})+4\theta(\theta^2G_{(2,4),i}+G_{(1,3),i})\right],$$

where $\theta^2=\sqrt{-1}$ and $\theta\in GF(l)$, l being the characteristic of F and

for $1\le i\le n$ and $1\le\beta\le 4$, $S_{(\beta),i}=\sum_{s\in C_{(\beta),i}} g^s$

for $1\le i\le n-2$, $1\le l,m\le 4$ and $l\ne m$, $G_{(l,m),i}=\frac{1}{2^{n-i+1}}\left(S_{(l),i}-S_{(m),i}\right)$

for $1\le i\le n$,

$$Y_i=\frac{1}{2^{n-i+1}}\left[\left\{1+\left(\sum_{j=i+1}^{n-2}\sum_{\beta=1}^{4}S_{(\beta),j}\right)+\left(S_{(1),n-1}-S_{(2),n-1}\right)+S_{(1),n}\right\}-\sum_{\beta=1}^{4}S_{(\beta),i}\right]$$

$$Y_0=\frac{1}{2^n}\sum_{t=0}^{2^n-1}g^t$$

Again S.K.Arora and Kulvir Singh describe the 8(n-2) primitive idempotents in the semisimple group algebra of the cyclic group G of order 2^n (n≥4) over the finite field F of prime power order q, where q=8k+1 is a quadratic residue modulo 2^n.

FC_{2^n} has 8(n-2) primitive idempotents given by

$$e_0^*=Y_0\ ,\quad e_{(1),n}^*=Y_1$$

$$e_{(1),n-1}^*=\frac{1}{2}\left[Y_2-2\theta^2(G_{(1,2),1}+G_{(3,4),1})\right],\ e_{(2),n-1}^*=\frac{1}{2}\left[Y_2+2\theta^2(G_{(1,2),1}+G_{(3,4),1})\right],$$

$$e_{(1),n-2}^*=\frac{1}{4}\left[Y_3-2\theta^2(G_{(1,2),2}+G_{(3,4),2})-4\theta(G_{(2,4),1}+\theta^2G_{(1,3),1})\right],$$

$$e_{(2),n-2}^*=\frac{1}{4}\left[Y_3+2\theta^2(G_{(1,2),2}+G_{(3,4),2})-4\theta(\theta^2G_{(2,4),1}+G_{(1,3),1})\right],$$

$$e_{(3),n-2}^*=\frac{1}{4}\left[Y_3-2\theta^2(G_{(1,2),2}+G_{(3,4),2})+4\theta(G_{(2,4),1}+\theta^2G_{(1,3),1})\right],$$

$$e_{(4),n-2}^*=\frac{1}{4}\left[Y_3+2\theta^2(G_{(1,2),2}+G_{(3,4),2})+4\theta(\theta^2G_{(2,4),1}+G_{(1,3),1})\right],$$

for $1\le i\le n-3$

$$e_{(1),i}^*=\frac{1}{8}\left[Y_{i+3}-2\theta^2(G_{(1,2),i+2}+G_{(3,4),i+2})-4\theta(G_{(2,4),i+1}+\theta^2G_{(1,3),i+1})-8\sqrt{\theta}(G_{(4,8),i}^*+\theta G_{(3,7),i}^*+\theta^2G_{(2,6),i}^*+\theta^3G_{(1,5),i}^*)\right],$$

$$e_{(2),i}^*=\frac{1}{8}\left[Y_{i+3}+2\theta^2(G_{(1,2),i+2}+G_{(3,4),i+2})-4\theta(\theta^2G_{(2,4),i+1}+G_{(1,3),i+1})+8\sqrt{\theta}(G_{(2,6),i}^*-\theta G_{(1,5),i}^*-\theta^2G_{(4,8),i}^*+\theta^3G_{(3,7),i}^*)\right],$$

$$e_{(3),i}^*=\frac{1}{8}\left[Y_{i+3}-2\theta^2(G_{(1,2),i+2}+G_{(3,4),i+2})-4\theta(G_{(2,4),i+1}+\theta^2G_{(1,3),i+1})+8\sqrt{\theta}(G_{(3,7),i}^*-\theta G_{(4,8),i}^*-\theta^2G_{(1,5),i}^*+\theta^3G_{(2,6),i}^*)\right],$$

$$e_{(4),i}^*=\frac{1}{8}\left[Y_{i+3}+2\theta^2(G_{(1,2),i+2}+G_{(3,4),i+2})+4\theta(\theta^2G_{(2,4),i+1}+G_{(1,3),i+1})-8\sqrt{\theta}(G_{(1,5),i}^*+\theta G_{(2,6),i}^*+\theta^2G_{(3,7),i}^*+\theta^3G_{(4,8),i}^*)\right],$$

$$e_{(5),i}^*=\frac{1}{8}\left[Y_{i+3}-2\theta^2(G_{(1,2),i+2}+G_{(3,4),i+2})-4\theta(G_{(2,4),i+1}+\theta^2G_{(1,3),i+1})+8\sqrt{\theta}(G_{(4,8),i}^*+\theta G_{(3,7),i}^*+\theta^2G_{(2,6),i}^*+\theta^3G_{(1,5),i}^*)\right],$$

$$e_{(6),i}^*=\frac{1}{8}\left[Y_{i+3}+2\theta^2(G_{(1,2),i+2}+G_{(3,4),i+2})-4\theta(\theta^2G_{(2,4),i+1}+G_{(1,3),i+1})-8\sqrt{\theta}(G_{(2,6),i}^*+\theta G_{(1,5),i}^*+\theta^2G_{(4,8),i}^*+\theta^3G_{(3,7),i}^*)\right],$$

$$e_{(7),i}^*=\frac{1}{8}\left[Y_{i+3}-2\theta^2(G_{(1,2),i+2}+G_{(3,4),i+2})+4\theta(G_{(2,4),i+1}+\theta^2G_{(1,3),i+1})-8\sqrt{\theta}(G_{(3,7),i}^*-\theta G_{(4,8),i}^*+\theta^2G_{(1,5),i}^*+\theta^3G_{(2,6),i}^*)\right],$$

$$e_{(8),i}^*=\frac{1}{8}\left[Y_{i+3}+2\theta^2(G_{(1,2),i+2}+G_{(3,4),i+2})+4\theta(\theta^2G_{(2,4),i+1}+G_{(1,3),i+1})+8\sqrt{\theta}(G_{(1,5),i}^*+\theta G_{(2,6),i}^*+\theta^2G_{(3,7),i}^*+\theta^3G_{(4,8),i}^*)\right],$$

Where $\theta^2=\sqrt{-1}$ and $\theta\in GF(l)$, l being the characteristic of F and

for $1\le i\le n$ and $1\le\beta\le 8$, $S_{(\beta),i}^*=\sum_{s\in C_{(\beta),i}^*} g^s$

for $1\le i\le n-3$, $1\le l,m\le 8$ and $l\ne m$, $G_{(l,m),i}^*=\frac{1}{2^{n-i+1}}\left(S_{(l),i}^*-S_{(m),i}^*\right)$

for $1\le i\le n-3$,

$$G_{(1,2),i} = G^*_{(1,2),i} + G^*_{(5,6),i},$$
$$G_{(1,3),i} = G^*_{(1,3),i} + G^*_{(5,7),i},$$
$$G_{(2,4),i} = G^*_{(2,4),i} + G^*_{(6,8),i},$$
$$G_{(3,4),i} = G^*_{(3,4),i} + G^*_{(7,8),i}$$

for $1 \le i \le n$,

$$Y_i = \frac{1}{2^{n-i+1}}\left[\left\{1+\left(\sum_{j=i+1}^{n-3}\sum_{\beta=1}^{8} S^*_{(\beta),j}\right)+\left(S^*_{(1),n-2}+\ldots+S^*_{(4),n-2}\right)+\left(S^*_{(1),n-1}-S^*_{(2),n-1}\right)+S^*_{(1),n}\right\}-\sum_{\beta=1}^{8} S^*_{(\beta),i}\right]$$

$$Y_0 = \frac{1}{2^n}\sum_{t=0}^{2^n-1} g^t$$

OTHER POSSIBILITIES:

Although, a number of codes have been found yet many problems exists for the primitive idempotents in the cyclic group algebra.One of the main problem is to find out the primitive idempotents for the cyclic group FG, G is cyclic group of order m [m=p^n or $p^n r^m$ or n(any natural number)], and F is Field of order q, where order of q modulo m [m=p^n or $p^n r^m$ or n(any natural number)] respectively is any number *t*(say) with $1 \le t \le \phi(m)$.

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