Elasto-Plastic analysis of thick-walled rotating cylinder subjected to combined loading

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Abstract: This investigation deals with analytical solution of elastic plastic stress in thick walled cylindrical shell subjected to internal pressure, rotation and radial thermal gradient. This analytical solution is obtained for thick walled cylinder by using Tresca criteria. Based on above criteria analytical solution is developed for single layer cylinder fixed at both ends which is known as plane strain condition. This elastic plastic analysis of thick walled cylinder is specifically very useful to design of mold of centrifugal casting machine. This solution is also helpful to design hollow cylindrical structure component for high temperature and high pressure application such as nuclear reactor, chemical plants, pressure vessels and oil and gas industry.

Index terms – Thick walled cylinders, rotation, internal pressure, temperature gradient.

1 Introduction

In many industrial applications, the cylinders are often subjected to extreme operating conditions characterized by high temperature, high pressure and/or corrosive environment. Most cylinders used in industry, are subjected to thermal loads produced by temperature variation in addition to mechanical loads and some of application also of cylinder also subjected to rotation like centrifugal casting.

Many authors have worked on elastic plastic analysis of cylinder subjected thermo-mechanical load, FE analysis of cylinder subjected to thermo-mechanical load. Several research studies have been conducted and reported in literature on the single layer cylinder and multilayer cylinder. Initially ,P.Montage and M.R.Horne [1] presented analysis for the axisymmetric behavior of a thin-walled cylinder with direction-fixed ends subjected to increasing radial pressure, the material being elastic-pure plastic and yielding according to the Tresca (maximum stress difference) criterion. T.Y. Chang and S.C. Chu [2] A numerical procedure based on the finite element method and incremental solution approach is presented for analyzing cylindrical pressure vessels deformed in the state of generalized plane strain. G.N.Brooks [3] presented Plasticity in shells is often contained near the ends of a segment where the bending stresses are significant. C.P.LUNGE and G.N.BROOKS [4] investigates the elastic-plastic behavior of a shallow spherical shell loaded radially through a flexible cylindrical nozzle. WANG ZHIQUM [5] paper the elastic-plastic fracture parameter-J-integral-for thick walled cylinder autofrettage is calculated using a nonlinear finite element method based on the flow theory of plasticity. S.LUKASIEWICZ and J.NOWINKA [6] deals with an elastic-plastic analysis of a cantilever cylindrical shell loaded at its free end by a concentrated radial force. YUSUF ORCAN and MUFIT GULGEC[7] Elastic plastic deformation through uniform internal energy generation is calculated using Tresca criteria and flow rules associated with it of a tube with free end. JOSEPH PERRY and JACOB ABOUDI [8] presented the optimal design of a modern gun barrel, there are two main objectives to be achieved: increasing its strength-weight ratio and extending its fatigue life. Eun-Yup Lee, Young-Shin Lee [10] this study focuses on the autofrettage process. It carrying capacity and fatigue lifetime of pressure vessels by increasing their residual stress. Mohammad zamani nejad, Abbas Rastgoo, Amin Hadi [11] present paper on an exact closed-form analytical solution for elasto-plastic deformations and stresses in a rotating disk made of functionally graded materials (FGMs) in which the elasto-perfectly-plastic material model is employed. S.M.KAMAL and U.S.DIXIT [12][13] have done study of thermal autofrettage of thick walled cylinders for plane stress and generalized plane strain condition. Vimal Patel [14] investigate on thermo elastic stress induced rotating multi-layer cylinder subjected to internal pressure and radial temperature gradient.

2 Analytical Solution

2.1 Cylinder fixed at both ends:

A thick-walled cylinder made of homogeneous and isotropic material with internal radius a and external radius b is considered. The internal wall of the cylinder is subjected to a temperature Tin and external wall is subjected to a temperature Tout. The cylinder is rotating with constant angular velocity ω and subjected to a uniform internal pressure Pi. Cylinder is loaded axis symmetrically in the radial direction and uniformly in the axial direction. For plane strain condition, strain in the axial direction (εz) is equal to zero,i.e.

$$\varepsilon_z = 0$$

The equation of equilibrium and strain-displacement relationship for the rotating cylinder under axisymmetric loading condition is written as,

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_t}{r} + \gamma\omega^2 r = 0 \quad (1)$$

$$\varepsilon_r = \frac{du}{dr} \quad \text{and} \quad \varepsilon_t = \frac{u}{r} \quad (2)$$

Above equation is hold both for elastic and plastic behavior.

Total strain also given by,

$$\varepsilon_r = \varepsilon_r^e + \varepsilon_r^p \quad (3)$$

$$\varepsilon_t = \varepsilon_t^e + \varepsilon_t^p \quad (4)$$

$$\varepsilon_z = \varepsilon_z^e + \varepsilon_z^p \quad (5)$$

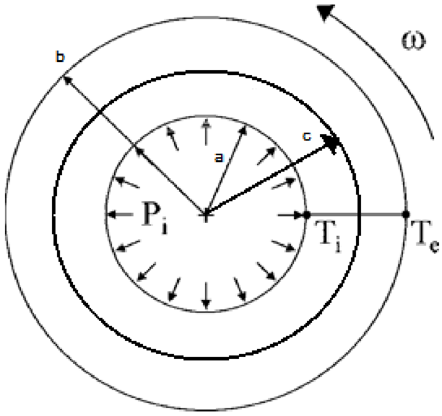

Figure 2.1 Configuration of cylinder subjected to internal pressure, rotation and radial Thermal gradient

2.1.1 Elastic Region ($c \le r \le b$):-

From Hook's consecutive law for tri axial stress in elastic filed including thermal load is given by,

$$\varepsilon_r^e = \frac{1}{E}(\sigma_r - v\,(\sigma_t + \sigma_z)) + \alpha T \quad (6)$$

$$\varepsilon_t^e = \frac{1}{E}(\sigma_t - v\,(\sigma_r + \sigma_z)) + \alpha T \quad (7)$$

$$\varepsilon_z^e = \frac{1}{E}(\sigma_z - v\,(\sigma_t + \sigma_r)) + \alpha T \quad (8)$$

From above equation (6), (7) and (8), Value of σ_r, σ_t and σ_z is given by,

$$\sigma_r = \frac{E}{(1+v)}\left(\frac{ve}{1-2v} + \varepsilon_r^e\right) - \frac{E}{(1-2v)}\alpha T \quad (9)$$

$$\sigma_t = \frac{E}{(1+v)}\left(\frac{ve}{1-2v} + \varepsilon_t^e\right) - \frac{E}{(1-2v)}\alpha T \quad (10)$$

$$\sigma_z = \frac{E}{(1+v)}\left(\frac{ve}{1-2v} + \varepsilon_z^e\right) - \frac{E}{(1-2v)}\alpha T \quad (11)$$

Using generalized Hook's law, strain compatibility (Eq. (2)), equilibrium equation (Eq. (1)), the strain in the elastic zone are given by

$$\varepsilon_r^e = \frac{1+v}{1-v}\,.\,\alpha.T - \frac{1+v}{1-v}\,.\,\alpha.\,\frac{1}{r^2}\int_c^r T.r.dr - \frac{(1+v)(1-2v)}{E(1-v)}\,\frac{3\gamma\omega^2 r^2}{8} + \frac{C_1}{2} - \frac{C_2}{r^2} \quad (12)$$

$$\varepsilon_t^e = \frac{1+v}{1-v}\,.\,\alpha.\,\frac{1}{r^2}\int_c^r T.r.dr - \frac{(1+v)(1-2v)}{E(1-v)}\,\frac{3\gamma\omega^2 r^2}{8} + \frac{C_1}{2} + \frac{C_2}{r^2} \quad (13)$$

$$\varepsilon_z^e = 0 \qquad \text{(Plain Strain condition - } \varepsilon_z = 0) \quad (14)$$

Substituting value of ε_r^e, ε_t^e and ε_z^e from (12), (13) and (14) into equations (9), (10) and (11)

And we get,

$$\sigma_r = C_1 + \frac{C_2}{r^2} - \frac{(3-2v)}{8(1-v)}\gamma\omega^2 r^2 - \frac{E\alpha\theta}{(1-v)} \quad (15)$$

$$\sigma_t = C_1 - \frac{C_2}{r^2} - \frac{(1+2v)}{8(1-v)}\gamma\omega^2 r^2 + \frac{E\alpha\theta}{(1-v)} - \frac{E\alpha T}{(1-v)} \quad (16)$$

$$\sigma_z = 2vC_1 - \frac{v}{2(1-v)}\gamma\omega^2 r^2 - \frac{E\alpha T}{(1-v)} \quad (17)$$

Where,

C_1 and C_2 are Integration constants.

$$\theta = \frac{1}{r^2}\int_a^r T.r.dr\ , \quad (18)$$

$$T = T_b + \frac{(T_a - T_b)\ln(\frac{b}{r})}{\ln(\frac{b}{a})} \quad (19)$$

2.1.2 Plastic Region ($a \le r \le c$):-

Yielding of material of cylinder begins according to Tresca criteria is given by,[13]

$$\sigma_r > \sigma_t > \sigma_z \quad (20)$$

$\sigma_r - \sigma_z = \sigma_y$ (21)

$\varepsilon_r^p = -\varepsilon_z^p$ And $\varepsilon_t^p=0$ (22)

Where, σ_y = Yield stress

$\sigma_z = \sigma_r - \sigma_y$ (23)

Now by equation (1),

$\sigma_t = \frac{d(r.\sigma_r)}{dr} + \gamma\omega^2 r^2$ (24)

From total strain equation (5), the generalized hook's law equation (8) and plane strain condition (14) axial plastic strain is given by,

$-\varepsilon_z^p = \varepsilon_z^e = \frac{1}{E}(\sigma_z - v\,(\sigma_t + \sigma_r)) + \alpha T$ (25)

From equation (22), (23), (24) and (25)

$-\varepsilon_z^p = \varepsilon_r^p = \frac{1}{E}\{(1\text{-}2v).\sigma_r - \sigma_y - v.r\frac{d\sigma_r}{dr} - v.\gamma\omega^2 r^2\} + \alpha T$ (26)

Total radial strain from equations (3),(6),(22-26) is written as,

$\varepsilon_r = \frac{1}{E}\{2.(1\text{-}2v).\sigma_r + (v-1)\sigma_y - 2v.r\frac{d\sigma_r}{dr} - 2v.\gamma\omega^2 r^2\} + 2\alpha T$ (27)

Aimed at ε_t, by equations (4),(7) and (22-24)

$\varepsilon_t = \frac{1}{E}\{(1\text{-}2v).\sigma_r + v\,\sigma_y + r\frac{d\sigma_r}{dr} + \gamma\omega^2 r^2\} + \alpha T$ (28)

By equation (2)

$\varepsilon_r = \frac{d(r.\varepsilon_t)}{dr}$

Put value from equation (27) and (28) in above equation and get

$\frac{d^2\sigma_r}{dr^2} + \frac{3}{r}\frac{d\sigma_r}{dr} - \frac{(1-2v)\sigma r}{r^2} = -\frac{\sigma_y}{r^2} - (3+2v)\gamma\omega 2 + \frac{E\alpha}{r^2}(T - r\frac{dT}{dr})$ (29)

Integrating above equation and we get,

$\sigma_r = \frac{c_3}{m}r^{-1+m} - \frac{c_4}{m}r^{-1+m} + \frac{1}{1-2v}\sigma_y - \frac{(3+2v)}{(7+2v)}\gamma\omega^2 r^2 + \frac{E\alpha}{2m}[(2-m)\theta_1 - (2+m)\theta_2]$ (30)

Where C_3 and C_4 are integration constants.

$\theta_1 = r^{-1+m}\int_a^r T r^{-m} dr$

$\theta_2 = r^{-1-m}\int_a^r T r^{m} dr$

$m = \sqrt{2(1-v)}$

By equation (30) and (24),

$\sigma_t = c_3 r^{m-1} + c_4 r^{-m-1} + \frac{\sigma_y}{1-2v} - \frac{2(1+2v)}{(7+2v)}\gamma\omega^2 r^2 + \frac{E\alpha}{2}[(2-m)\theta_1 - (2+m)\theta_2 - 2T]$ (31)

And axial stress is given by equation (23), i.e.

$\sigma_z = \sigma_r - \sigma_y$

Yielding sets in at r=a according to $\sigma_y = \sigma_r - \sigma_z$. In the elastic plastic state the cylinder is composed of the plastic region for $a \le r \le c$ and the elastic region for $c \le r \le b$. Altogether there are five unknowns consisting of elastic plastic interface radius c and the integration constants C_1 C_2 ,C_3 and C_4 which are function of load parameter.

Using the condition that σ_r=0 at r=b and σ_r= -p at r=a

And the three no redundant continuity conditions of σ_r , σ_t and σ_z at the interface r=c, the constants are determined as,

$C_1 = -\frac{C_2}{b^2} + \frac{E\alpha\theta_b}{1-v} + \frac{(3-2v)}{8(1-v)}\gamma\omega^2 b^2$ (32)

$C_2 = \{\frac{(bc)^2}{b^2 - c^2(1-2v)}\} \times \{\frac{\gamma\omega^2}{8(1-v)}[(3-6v)c^2\text{-}(3\text{-}2v)(1\text{-}2v)b^2] + \sigma_y + \frac{E\alpha}{1-v}[\theta_c - T_c - (1-2v)\theta_b]\}$ (33)

$C_3 = \left(c_4\, a^{-1-m} + \frac{m}{1-m^2}\sigma_y + \frac{m(3+2v)}{(7+2v)}\gamma\omega^2 a^2 - pm\right) \times a^{1-m}$ (34)

$C_4 = \{\frac{1}{1-2v}\sigma_0[(1+m)\left(\frac{c}{a}\right)^{-1+m} - 2 - \frac{1}{v}\left(\frac{c}{a}\right)^{-1+m} - \frac{1}{v}] + \gamma\omega^2 c^2[(1+m)\frac{(3+2v)}{(7+2v)}\left(\frac{c}{a}\right)^{-3+m} - \frac{(5+6v)}{(7+2v)} + \frac{1}{2(1-v)} - \frac{1}{v}\frac{(3+2v)}{(7+2v)}(\left(\frac{c}{a}\right)^{-1+m} - 1)] + \frac{E\alpha}{2vm}[(v+vm-1)(2-m)\theta_{1c} - (v+vm+1)(2+m)\theta_{2c}] - \frac{(v+1)E\alpha Tc}{v} + \frac{p}{v}\left(\frac{c}{a}\right)^{-1+m}[1\text{-}v(1\text{+}m)] + \frac{\sigma_y}{v} - \gamma\omega^2 c^2\frac{1}{1-v}\} \times \frac{1}{R}$ (35)

Where,

$R = \frac{c^{-1-m}}{vm}[\left(\frac{c}{a}\right)^{2m} - 1] - \frac{c^{-1-m}}{m}[(1+m)\left(\frac{c}{a}\right)^{2m} + m - 1]$

Using equations (32)-(35), remaining unknown c is obtained in the solution of the nonlinear equation written for the continuity of σ_r at the interface r=c as a function of load parameter.

3 Result and Discussion

3.1 Design of mold

As discussed earlier, mold of centrifugal casting machine is subjected to centrifugal body force due to rotation of the mold, internal pressure created by the molten metal on the inner wall of the mold and thermal load due to temperature gradient through wall thickness of the mold. Using the solution obtained in the section 2, stress distribution for the mold is obtained with mold dimension and loading condition as presented in the Table 3.1.

Mold Dimensions	
Inner radius(a) (mm)	1000
Outer radius(b) (mm)	1200
Thickness (mm)	200
Length (mm)	1000
Loading conditions	
Internal pressure P_i (Mpa)	2.3
Angular velocity ω (RPM)	405
Inner wall temperature T_{in} (C)	700
Outer wall temperature T_{out} (C)	200

Table 3.1Mold dimension and loading condition

A medium carbon low alloy steel AISI 4340 with material properties as presented in table 3.2 and 3.3 is defined as material for the cylinder.

Material	Fe	C	Mn	Si	Cr	Ni	Mo	P	S
AISI 4340	95.4	0.4	0.75	0.25	0.9	2.0	0.25	0.025	0.025

Table 3.2 Chemical composition of AISI 4340 steel

Density(kg/m^3)	Modulus of elasticity(GPa)	Poisson's ratio (v)	Thermal expansion coefficient(/c)	Thermal conductivity(W/m.K)	Yield strength (Mpa)
7720	184	0.3	16.4 x 10^{-6}	367	702

Table 3.3 Mechanical and thermal properties of AISI 4340 Steel

3.2 Analytical Result

- Figure 3.1 shows radial, tangential and axial stress distribution from internal radius to external radius of the mold for internal pressure of 2.3 MPa, angular velocity of 405 RPM, internal wall temperature of 400 °C and external wall temperature of 150 °C.for analytical solution.
- Figure shows that initial at internal radius axial stress is higher compare to all stress and at outer radius hoop stress have higher value
- At inner radius radial stress have internal press value while at outer radius it has zero stress.

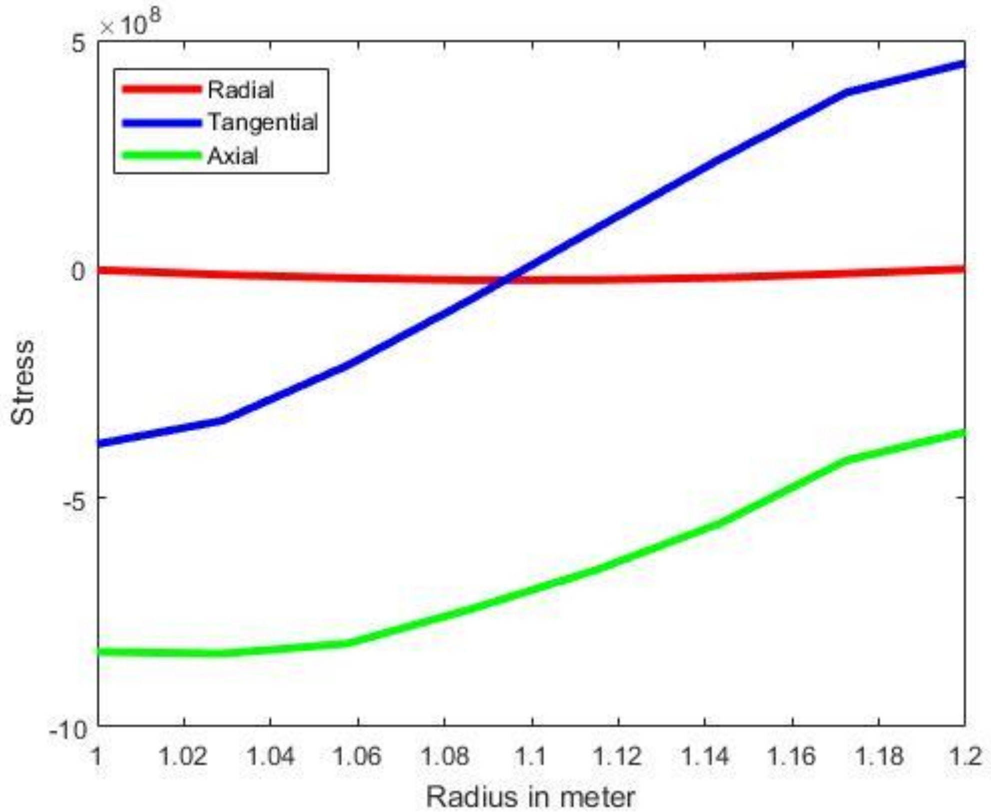

Fig. 3.1 Analytical Stress distribution for mold subjected to internal pressure = 2.3 MPa, Angular velocity = 405 RPM, inner wall temperature =400 °C and outer wall temperature = 150 °C

3.3 FEM solution:

A three dimensional axisymmetric model of cylinder with the internal radius 1 meter, external radius 1.2 meters and length 1 meter is developed. A coupled thermo mechanical approach is used for the analysis of a cylinder. A medium carbon low alloy steel AISI 4340 with material properties as presented in table 3.1,3.2 and 3.3 is defined as material for the cylinder.

- Figure 3.2 shows radial, tangential and axial stress distribution from internal radius to external radius of the mold for internal pressure of 2.3 MPa, angular velocity of 405 RPM, internal wall temperature of 400 °C and external wall temperature of 50 °C.for analytical solution.
- Figure shows that initial at internal radius axial stress is higher compare to all stress and at outer radius hoop stress have higher value
- At inner radius radial stress have internal press value while at outer radius it has zero stress.

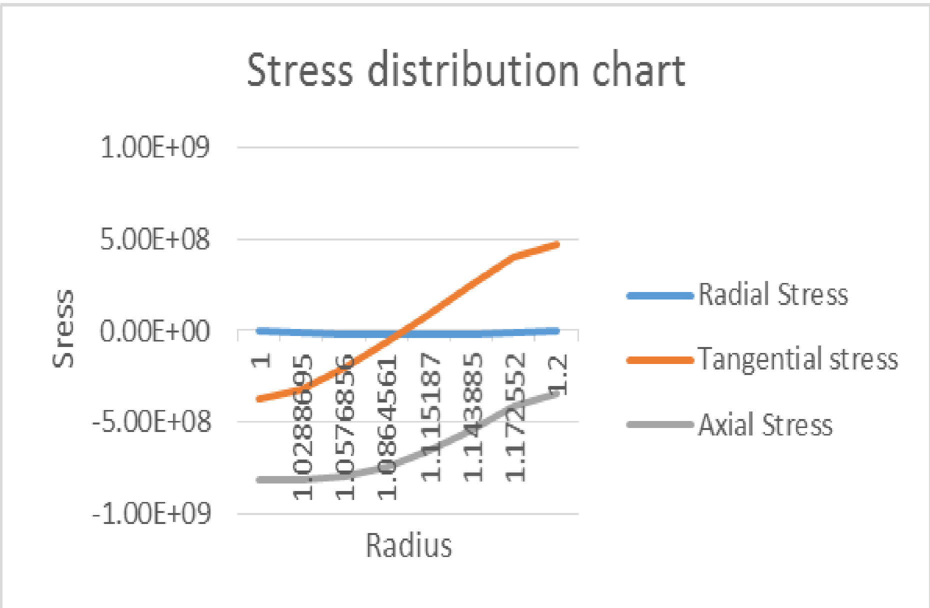

Fig. 3.2FEM Stress distribution for mold subjected to internal pressure = 2.3 MPa, Angular velocity = 405 RPM, inner wall temperature =400 °C and outer wall temperature = 150 °C

- Both figure 3.1 and 3.2 shows that both solution have very good agreement with each other

The results shown in this investigation are just for a particular loading combinations and for single material selected. However, analytical solution (stress distribution) for any combination of loading and material properties can be obtained.

4 Conclusion

The main purpose of the current study is to derive analytical solution for rotating thick walled cylinder subjected to internal pressure and radial thermal gradient. Solution is utilized to obtain the stress distribution of mold subjected to each individual load and combined load in order to determine critical load for the design of mold. The results obtained in the current investigation lead to the following conclusions.

- The combined effects of rotation, internal pressure and temperature must be taken into account when designing the thick walled cylinder especially for centrifugal casting application, to ensure their maximum efficiency and maximum availability.
- Effect of thermal load is dominant over the load due to internal pressure and rotation. Thick walled cylinder is subjected to combined load. The tangential stress is more affected by the variation of the temperature then the internal pressure value.

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