Laplacian Centrality in Transportation Network

Tulsi Bora[1] , Arun Kumar Baruah[2]

Asstt. Professor[1] , Professor[2]

Deptt. of Mathematics ,CNB College, Bokakhat[1]. Deptt. of Mathematics, Dibrugarh University[2]

ABSTRACT

The centrality of vertices has been a key issue in network analysis. In this work we propose a centrality measure for networks, which we refer to as Laplacian centrality, that provides a general framework for the centrality of a vertex. This centrality based on the idea that the importance of centrality of a vertex is related to the ability of the network to respond to the deactivation or removal of that vertex from the network. In particular, the Laplacian centrality of a vertex is defined as the relative drop of Laplacian energy caused by the deactivation of this vertex. The Laplacian energy of network G with n vertices is defined as $E_L(G) = \sum_{i=1}^{n} \lambda_i^2$ where λ_i is the eigenvalue of the Laplacian matrix of G. That is, compared to other standard centrality measures. Laplacian centrality is an intermediate measuring between global and local characterization of the importance of centrality of a vertex. We further investigate the validness and robustness of the Laplacian centrality measure by illustrating this method to transportation network data sets and obtain reliable results, which provide strong evidences of the measure's utility.

Key words : Network, Centrality, Laplacian energy, Laplacian centrality, Deviation.

1. Introduction

The use of Social Network Analysis (SNA) to understand complex network demonstrates the importance of the implicit connections within groups that arise from day to day social activity. Now a days it is the most important to observing, detecting and analyzing transportation networks in understanding who is central to the functionality of these groups that form around the common goal of engaging in transportation activities. As a result, for both academic and practical reasons ,researchers and analysts from several areas have a highly interest in understanding the centrality within networks.

SNA provides us tools for mapping and measuring relationships and flows between people, groups, organizations, computers, and many other connected bits of information/knowledge. The vertices in the network represent people and groups while the edges show relationships, connections, or flows between the vertices. SNA provides both a visual and a mathematical analysis of these interrelationships. Recent studies of networks in political science range from such diverse topics as international conflict [1], terrorism [2] and policy networks [3] to disciplinary introspection about job placement in political science [4].

To understand network structure, we evaluated of their location relative to all other actors in the network. For networks, the most readily examined measure of location means how close is the object to the center, or centrality. Finding the important vertices with high centralities in order to characterize the properties of the networks has significant uses in many fields. These include synchronization transition, the spread of epidemics, and the transmission of information. For example, in diffusive systems the vertices with large degree play a crucial role, which are decisive in resolving the traffic jam at a bottleneck [5]. Jackson[6] analyzed the different aspects of the positioning of the nodes with the help of different centralities. The degree method has an advantage and disadvantage also. It is calculated only the local structure around a vertex. A vertex is connected to many other vertices , it might not be in a position to reach others quickly to access information or flow[7]. If the network consists of more than two communities ,the nodes in the smaller community would exhibit lower "subgraph" centrality ranks. So, if species to community relation are to be known in food web[8,9] an intermediate characterization of vertex centrality has been claimed as a necessity for the study.

The centrality of vertices, or the identification of vertices which are more ''central'' than others, has been a key issue in network analysis. Various centrality measurements are used viz. degree centrality, closeness centrality, betweenness centrality, eigenvector centrality and subgroup centrality.

In this paper, we propose a new centrality strategy for transportation networks, which permits one to consider more ''intermediate'' environmental information around a vertex is called ''Laplacian centrality method'' because it is from the use of a matrix valued function that describes the so-called ''Laplacian energy'' of the network. The basic idea is that the importance of centrality of a vertex is related to the ability of the network to respond to the deactivation of the vertex from the network. In particular, the relative drop of Laplacian energy in the network caused by the deactivation of this vertex from the network will be used as the indicator to show its importance in the network. We further investigate the validity and robustness of this new measure by applying this method to some classical data sets of social networks. Successful applications on those bench mark data sets are evidences of the utility of this proposed centrality measurement.

This paper is organized as follows. In Section 2, we give some useful graph theory notations and terminology. In Section 3, we present the definition of Laplacian centrality. In Section 4, we give a theorem to show a structural description of the Laplacian centrality. Analytical and numerical results based on various centrality measures applying on Dispur Transportation Network in Section 5. In section 6 and 7, we focus the analysis and results of the proposed Network. Conclusions are made in Section 8.

2. Graph Theory Notation and Terminology

Social network usually represented as a graph. The vertices are the individuals, and the edges represent the social links. In this paper, we consider the symmetric case where transportation networks are represented by undirected graphs. Multiple edges are two or more edges connecting the same two vertices. Graphs with multiple edges are called multigraph. A degenerate edge of a graph which joins a vertex to itself, is called a loop. The number of edges that are incident to a vertex is called the degree of the vertex. The neighborhood of a vertex v is the set of all vertices adjacent to v. Graph entropy measures are always used for determining the structural information content of graphs, which has been proved to play an important role in a variety of problem areas, including biology, chemistry, and sociology [4]. Laplacian energy, which could be thought as one kind of graph entropy, representing a certain coherent measuring of a network, is used here to measure the importance of centrality of a vertex by the relative drop of Laplacian energy in the network caused by the deactivation of this vertex from the network. In Section 3, we will introduce the definition of Laplacian energy of a network and Laplacian centrality of a vertex.

Let G be an undirected graph, consisting of a set of n vertices $V(G)= \{v_1, v_2, \ldots, v_n\}$ and a set of m edges. The number of edges that are incident to a vertex is called the degree of the vertex. Let $A(G) = (a_{i,j})_{n\times n}$ be the adjacency matrix of the graph G, where the element $a_{i,j}$ equals 1 if there is an edge between vertices i and j , and 0 if there is not.

3. Laplacian Centrality

In the following, we will first introduce the definition of Laplacian matrix and Laplacian energy for a graph, then define the Laplacian centrality for a vertex.
Let G be a simple graph without graph loops or multiple edges of vertices, and

$$D(G) = \mathrm{diag}(d_1, d_2, \ldots, d_n) = \begin{pmatrix} d_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & d_n \end{pmatrix}$$

be the diagonal matrix with the vertex degrees $d_1, d_2, \ldots, d_n$ of its vertices $v_1, v_2, \ldots v_n$. Define $L(G) = D(G) - A(G)$ as the Laplacian matrix of the graph G.

Definition 1 *If G is a graph of n vertices, and $\lambda_1, \lambda_2, \ldots \lambda_n$ are the eigenvalues of its Laplacian matrix. The Laplacian energy of G is defined as the following invariant:*

$$E_L(G) = \sum_{i=1}^{n} \lambda_i^2$$

Lemma 1 [10] *For any graph G on vertices with vertex degrees $d_1, d_2, \ldots d_n$, we have*

$$E_L(G) = \sum_{i=1}^{n} (d_i^2 + d_i)$$

Lemma 2 [10] *If H is an arbitrary subgraph of a graph G, then $E_L(H) \leq E_L(G)$.*

Definition 2 *If G is a graph on n vertices* $\{v_1, v_2, \ldots, v_n\}$. *Let H be the graph obtained by removing vertex*

v_i from G . The Laplacian centrality C_i^L of vertex v_i is defined as

$$C_i^L = (\Delta E)_i = E_L(G) - E_L(H)$$

4. Calculation of Laplacian Centrality, a Graph Theory Result

4.1. Graph Theoretical Descriptions

Theorem 1 [11] *If G is a graph of n vertices, then the Laplacian centrality with respect to v is*

$$C_v^L = (\Delta E)_v = d_G^2(v) + d_G(v) + 2 \sum_{v_j \in N(v)} d_G(v_i)$$

where $N(v)$ is the set of neighbors of v in G and $d_G(v_i)$ is the degree of v_i in G.
From Theorem 1, we notice the following facts:
Firstly, the Laplacian centrality agrees with the standard measures on assignment of extremes. For example, it gives the maximum value to the central vertex of a star, and equal value to the vertices of a cycle or a complete graph.
Secondly, we know that the degree centrality is the number of vertices which can be reachable from v directly. The Laplacian centrality of a vertex involves the information of vertices that can be reachable to v within two steps and as a result the Laplacian centrality of a vertex takes not only the local environment around it into account but also the larger immediate environment around its neighbors. Therefore it is an intermediate measure between global and local characterizations of the position of a vertex within networks. Because of this we should anticipate that it will reveal differences in network structure that emerge out of significant local influence upon areas of the graph.

4.2. Comparison with Local and Global Centrality Methods

In this section, we will give a simple example to show the differences between degree method and Laplacian methods with the popular existing centrality measures respectively.

Please see **Figure 1**, based on degree centrality, node no. 3 has higher ranking than node no. 15 because the degree of node no. 3 is 4 while the degree of node no. 15 is 3.
But based on Laplacian method, would have higher ranking than because

$$(\Delta E)_3 = d_G^2(3) + d_G(3) + 2 \textstyle\sum_{v_j \in N(v)} d_G(v_i) = 36$$

$$(\Delta E)_{15} = d_G^2(15) + d_G(15) + 2 \textstyle\sum_{v_j \in N(v)} d_G(v_i) = 42$$

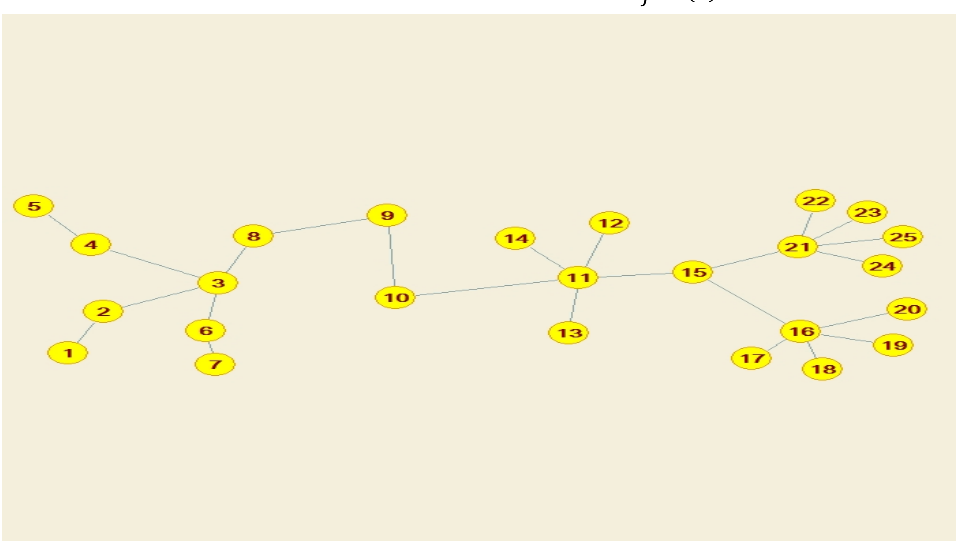

Figure 1. Example for comparison of degree centrality and Laplacian centrality method**.**

5. Application and Experimental Results

Dispur is the largest city in Assam and one of the fastest developing cities in India. With the rapid growth of population in the city, the road traffic problems are also increasing at an alarming rate. The development of a city or town leads to the growth of the number of vehicles which is directly linked to increased traffic congestion and a growing number of accidents and fatalities. Road traffic problems like congestion, unpredictable travel-time delays and road accidents are taking a serious shape in the city. The main objective of this study is to analyze the potential of bridging centrality on transportation network ,viz. Dispur city map. It is a well planned city and capital of Assam. We take 35 major bus stoppages of this Dispur area to analyze the bridging nodes .

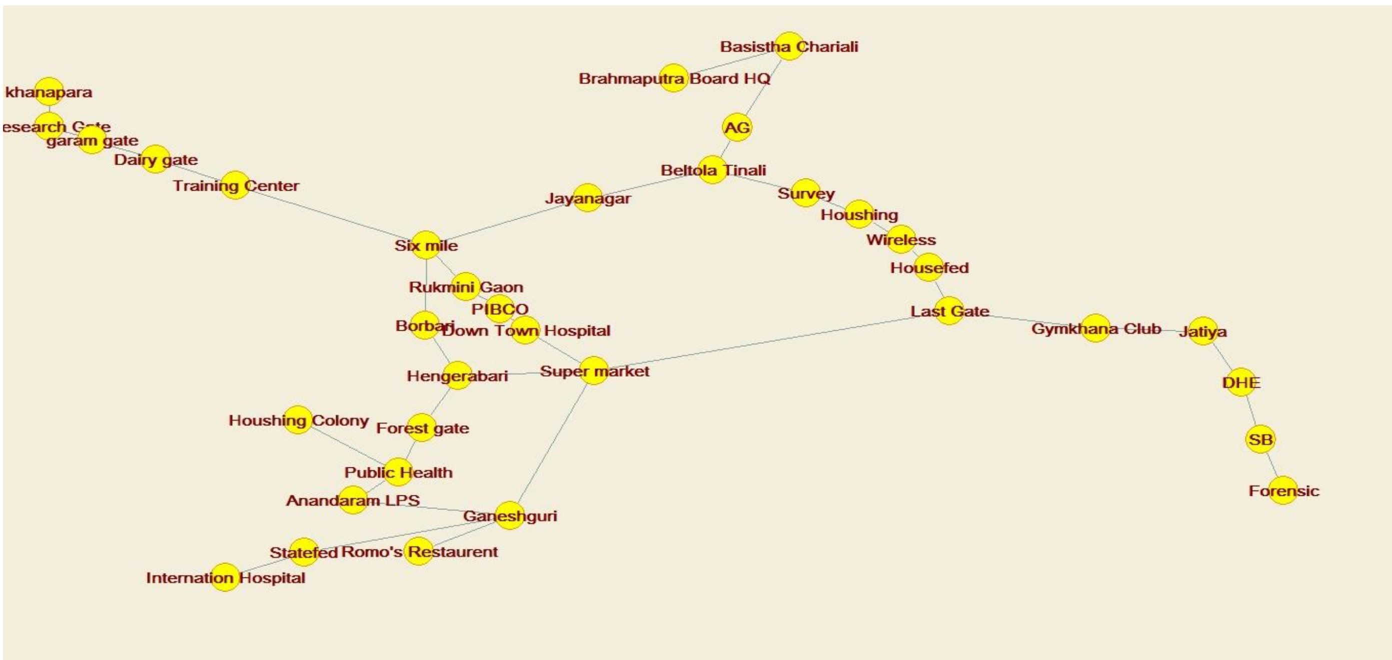

Figure 2. Major bus stoppage of Dispur city.

The information provided by the four standard centrality measures and our Laplacian centrality measure as applied to the 35 major bus stoppages of the transportation network directly involved in the operation are provided in **Table 1**, where the centrality scores are normalized which is dividing by the highest score of each method. We also list the rank for each stoppages. Note that frequently actors will exhibit the same scores for a number of measures. In the case when ties occur, we usually assign them the same rank.

6. Consensus Rank

In this research project, we used the similar ideas and principles of consensus applied in bioinformation in the ranking study of social networks. The consensus ranks are calculated as follows. At first, for each stoppages, we calculate the mean of its five ranks from various methods. For example, Ganeshguri gets ranks {1,6,7,2,2} from five centrality methods respectively, thus its mean of rank is 3.6 (a fractional number); secondly, we sort the mean values of these 35 stoppages from smallest to largest, the output of order is defined as the consensus rank of these 35 stoppages. Note that when ties happen (*i.e.*, more than one stoppages have the same mean value) we follows the same criterion as used in finding the rank in transportation network. We list the consensus rank of these 35 stoppages at the last column of **Table 1**.

Id	Label	Scores					Ranks					Consensus
		Degree	Betweenness	Closeness	Laplacian	Eigen vector	Degree	Betweenness	Closeness	Laplacian	Eigen vector	
1	khanapra	0.25	0	0.450002	0.136364	0.063705	31	30	34	32	34	34
2	Research Gate	0.5	0.130953	0.510123	0.272727	0.118036	8	22	31	27	31	29
3	Garm gate	0.5	0.253968	0.583332	0.318182	0.164833	8	13	28	20	27	25
4	Dairy gate	0.5	0.369047	0.673797	0.318182	0.228099	8	11	23	20	23	22
5	Training Center	0.5	0.476191	0.787501	0.409091	0.362492	8	8	13	11	14	9
6	six mile	1	0.94246	0.933335	0.818182	0.670963	1	2	4	3	4	2
7	Rukmini gaon	0.5	0.101432	0.834439	0.409091	0.393571	8	28	9	11	12	15
8	PIBCO	0.5	0.103175	0.840002	0.318182	0.340618	8	27	8	20	17	17
9	Down town hospital	0.5	0.126984	0.875	0.409091	0.500455	8	26	6	11	7	10
10	Super market	1	1	1	1	1	1	1	1	1	1	1
11	Ganeshguri	1	0.505953	0.857146	0.863636	0.799774	1	6	7	2	2	4
12	Statefed	0.5	0.130953	0.707865	0.363636	0.358477	8	22	21	15	15	18
13	International Hospital	0.25	0	0.597155	0.136364	0.139676	31	30	27	32	30	32
14	Houshing colony	0.25	0	0.605772	0.181818	0.164855	31	30	26	31	26	31
15	Public Health	0.75	0.168652	0.720002	0.5	0.414082	4	21	19	7	11	12
16	forest gate	0.5	0.192461	0.818182	0.409091	0.435126	8	20	11	11	9	11
17	Hengerabari	0.75	0.781747	0.992197	0.636364	0.733594	4	3	2	4	3	3
18	Borbari	0.5	0.65873	0.954547	0.454545	0.531636	8	5	3	8	6	6
19	Gymkhana club	0.5	0.476191	0.77778	0.363636	0.34052	8	8	14	15	18	13
20	Jatiya	0.5	0.369047	0.666667	0.318182	0.216572	8	11	24	20	24	23
21	DHE	0.5	0.253968	0.577981	318182	0.160261	8	13	29	20	28	26
22	SB	0.5	0.130953	0.506024	0.272727	0.116606	8	22	32	27	32	30
23	Forensic	0.25	0	0.446808	0.136364	0.063355	31	30	35	32	35	35
24	Last gate	0.75	0.779761	0.919708	0.636364	0.642732	4	4	5	5	5	5
25	Housefed	0.5	0.291666	0.812905	0.363636	0.345265	8	13	12	15	16	14
26	Wireless	0.5	0.242063	0.750001	0.318182	0.234753	8	17	16	20	22	21
27	Houshing	0.5	0.218253	0.724138	0.318182	0.216392	8	18	17	20	25	24
28	Survey	0.5	0.214286	0.720002	0.363636	0.26277	8	19	19	15	20	18
29	Beltola Tiniali	0.75	0.498016	0.759036	0.545455	0.392106	4	7	15	6	13	7
30	AG	0.5	0.253968	0.646155	0.363636	0.238058	8	13	25	15	21	20
31	Basistha Chariali	0.5	0.130953	0.557521	0.272727	0.146074	8	22	30	27	29	28
32	Brahmaputra board HQ	0.25	0	0.486486	0.136364	0.072265	31	30	33	32	33	33
33	Jayanagar	0.5	0.430556	0.823529	0.454545	0.427158	8	10	10	8	10	8
34	Romo's restaurent	0.25	0	0.700001	0.227273	0.299941	8	30	22	30	19	27
35	Anandaram LPS	0.5	0.087302	0.724138	0.454545	0.464796	8	29	17	8	8	16

Table 1. The centrality scores based on five methods for 35 vertices in Dispur Network; their ranks and consensus rank in all 35 vertices.

7. Deviation of Each Method from Consensus Result

To further analyze, we use the following "deviation" score to numerically evaluate the distance from the output based on each centrality method to the consensus rank:

$$deviation(C) = \sum_{i=1}^{35}(rank^{C}\,(i) - consensus(i))^2$$

where $rank^C(i)$ is the rank of i-th stoppages based on centrality measurement C (C ϵ {Laplacian, degree, closeness, betweenness, eigenvector centrality}), and $consensus(i)$ is the consensus rank of i-th stoppage.

Clearly, the smaller the deviation, the better is the output. A method with the smallest deviation is regarded as the one with "the best fit". The deviations for all centrality methods are presented at the last two rows of **Table 2**, which shows that Laplacian method has the smallest deviation. That is, Laplacian centrality method has the best fit to the consensus ranking results, which is a further evidence of its effectiveness and reliability to identify major players in social network.

	Degree	Betweenness	Closeness	Laplacian	Eigen vector
Deviation	3877	1802	511	306	317

Table 2. Deviation of five centrality methods of all 35 vertices.

8. Concluding Remarks

In this final section, we survey some properties of the new method which is balanced global/local measurement qualities, its accuracy and effectiveness, efficiency, and its future applications. The review is conducted from several different angles viz. graph structure, verification of known facts from experimental testing, and the consensus for comparison.

From the graph theoretical point of view, graph centrality measurements can be roughly classified as two types: local or global. Degree centrality is a typical example of local measurement which is consider the information of the number of vertices that are reachable from directly, while betweeness, closeness and eigenvector, are more global. Laplacian centrality reveals more connection information beyond its immediate surrounding neighborhood, thus serves as an intermediate between global and local characterizations.

Centrality measurement based on shortest path calculations such as, betweeness, closeness, etc. are powerful tools for the detection of bottlenecks in networks the cut-vertices of connected graphs. Consequently, these types of vertices are scored more favorably if global connectivity is relatively low. In these lower connectivity examples, Laplacian centrality will provide a more balanced measurement, which takes both bottleneck information and local density information into account. As we discussed above, from a graph theoretical point of view different methods reveal different measurements of importance due to their different structural emphasis. Applying the new method in this transportation network the Laplacian method provides a better performance as a network analysis tool. This is supported by two different approaches: confirmation of analyses based on known intelligence information, and consensus comparison, where the study shows that the Laplacian method has the smallest deviation from the consensus result, providing additional evaluation of the reliability of Laplacian centrality.

REFERENCES

[1] Z. Maoz, D. Kuperman, L. Terris and I. Talmud, "Structural Equivalence and International Conflict: A Social Networks Analysis," *Journal of Conflict Resolution*, Vol. 50, No. 5, 2006, pp. 664-689. doi:10.1177/0022002706291053

[2] M. Sageman, "Understanding Terror Networks," Philadelphia, University of Pennsylvania Press. 2004.

[3] J. T. Scholz, R. Berardo and B. Kile, "Do Networks Enhance Cooperation? Credibility, Search, and Collabora-tion," *Journal of Politics*, Vol. 70, No. 2, 2008, pp. 393-406.

[4] J. Fowler, B. Grofman and N. Masuoka, "Social Net-works in Political Science: Hiring and Placement of Ph. Ds, 1960-2002. Political Science: 2007, pp. 727-739.

[5] S. Sreenivasan, R. Cohen, E. Lopez, Z. Toroczkai and H. E. Stanley 2007. Phys. Rev. E 75, 036105. doi:10.1103/PhysRevE.75.036105

[6] M. O. Jackson, "Presenting and Measuring networks," Chapter 2, *Social and Economic Networks*, Princeton University Press, Princeton, 2008.

[7] T. Opsahl, F. Agneessens and J. Skvoretz, "Node Centrality in Weighted Networks: Generalizing Degree and Shortest Paths," *Social Networks*, Vol. 32, No. 3, 2010, pp. 245-251. doi:10.1016/j.socnet.2010.03.006

[8] F. Jordan and I. Scheuring, "Network Ecology: Topological Constraints on Ecosystems Dynamics," *Physics of Life Reviews*, Vol. 1, No. 3, 2004, pp. 139-172. doi:10.1016/j.plrev.2004.08.001

[9] F. Jordan, W. C. Liu and A. J. Davis, "Topological Key-stone Species: Measures of Positional Importance in Food Webs," *Oikos*, Vol. 112, No. 3, 2006, pp. 535-546. doi:10.1111/j.00301299.2006.13724.x

[10] M. Lazic, "On the Laplacian Energy of a Graph," *Czechoslovak Mathematical Journal*, Vol. 56, No. 131, 2006, pp. 1207-1213.

[11]Xingqin Qi., et. Al., " Terrorist Networks, Network Energy and Node Removal : A New Measure of Centrality Based on Laplacian Energy." *Social Networking*, 2013, 2, 19-31, http://dx.doi.org/10.4236/sn.2013.21003.