Tauberian-Type Theorems with Application to the Fourier-Stieltjes Transformation

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Abstract: In this paper Fourier-Stieltjes transform is extended in the distributional generalized sense. Tauberian-type theorems for the distributional Fourier-Stieltjes transform are proved.

Index Terms: Stieltjes transform, Fourier transform, Fourier-Stieltjes transform, generalized functions, quasiasymptotic behavior.

1. Introduction

Tauberian theory, since its inception in the early 20th century through the pioneering work of Hardy and Littlewood, has played an important role in many areas of mathematics, including summability theory, partial and ordinary differential equations, number theory and harmonic analysis. One of the most important applications of Tauberian theory is the proof of prime number theorem. Furthermore, a direct application of the Wiener-Tauberian theorem yields various Weyl’s laws, by which we mean an estimate of the counting function of eigenvalues of certain differential operators.

The name “Tauberian” is typically used for the theorems that allow conclusions about asymptotic behavior of the functions themselves or some other averages of them from the behavior of certain averages of (generalized) functions. The name is also applied to theorems that relate the asymptotic behavior of a function to asymptotic behavior of its Laplace, Fourier, or other integral transform. In this paper we give definition of the space $FS_{\alpha}$, conventional Fourier-Stieltjes transform distributional Fourier-Stieltjes transform. This enables us to obtain the Tauberian-type theorems of quasiasymptotic behavior of distributions at infinity. We give sufficient conditions under which the behavior at infinity of distributional Fourier-Stieltjes transformation.

Notation1.1. As usually $R$, $C$, $N$ are the spaces of real, complex and natural numbers; $N_0 = N \cup \{0\}$. $D$ is the space of infinitely differentiable functions with compact support. $S'$ denotes the space of tempered distributions with support in the $[0, \infty)$. The space of rapidly decreasing functions is denoted as $S$, $S'$ is the space of all distributions of slow growth. $F(s, y)$ denotes distributional Fourier-Stieltjes transform.

A positive continuous function $L$ defined on $(0, \infty)$ is called slowly varying function at $\infty$ if for every $k > 0$,

$$
\lim_{k \to \infty} \frac{L(kx)}{L(k)} = 1 \tag{1.1}
$$

We denote by $\sum_{\epsilon}$ the set of all slowly varying functions at $\infty$. For the properties of slowly varying functions, we refer the reader to [6]

If $L$ is slowly varying function at $\infty$, then for every $\epsilon > 0$, there $A_\epsilon > 0$ such that

$$
x^{-\epsilon} < L(x) < x^{\epsilon} \quad \text{When } x > A_\epsilon \tag{1.2}
$$
Recall that for $\alpha > 0$, $x_+^\alpha = H(x)$, where $H$ is a Heaviside’s function. The following scale of distributions from $S'\ $ has been used in the investigation of quasiasymptotic behavior of distributions:

$$f_{\alpha+1} = \begin{cases} 
\frac{Ht^\alpha}{\Gamma(\alpha+1)}, & \alpha > -1 \\
D^\alpha f_{\alpha+1}, & \alpha \leq 1, \alpha + n > -1
\end{cases}$$

(1.3)

Where $D$ is the distributional derivative.

2. Definitions

Definition 2.1. The quasiasymptotic behavior of distribution (q.a.b.) at infinity.

If $T$ is distribution from $S'\ $ such that the distributional limit

$$\lim_{k \to \infty} \frac{L(kx)}{\rho(k)} = \gamma(x)$$

(2.1)

exists in $S\ $ ($\gamma(x) \neq 0$), then $T$ is called the quasiasymptotic behavior at infinity related to the regular function $\rho(k)\ = k^\alpha L(k)$ with the limit $\gamma$; write this as

$$T \sim \gamma \quad \text{in } S' \quad \text{as } x \to \infty$$

(2.2)

Here $\rho$ is regularly varying at infinity and the limit $\gamma$ from $S'\ $ is of the form

$$\gamma(x) = \mathbb{C} f_{\alpha+1}(x)$$

(2.3)

We will repeat some well-known facts about the quasiasymptotic behavior from [8]

Let $f \in S'\ $. It is said that $f$ has the q. a. b. at $\infty$ with the limit $g \neq 0$ with respect to

$$k^\alpha L(k),\ L \in \left\{ \left(1/k\right)^\alpha L\left(1/k\right),\ L \in \sum_{0}^{1}, \alpha \in R, \right\} \right.$$  

(2.4)

If

$$\lim_{k \to \infty} \left\langle \frac{f(kt)}{k^\alpha L(k)}, \phi(t) \right\rangle = \left\langle g(t), \phi(t) \right\rangle, \phi \in S$$

Definition 2.2 A function $\rho (a, \infty) \to R$, $a \in R$, is called regular varying function at infinity if it is positive, measurable, and there exists a real number $\alpha$ such that for each

$$x > 0, \lim_{k \to \infty} \frac{\rho(kx)}{\rho(k)} = x^\alpha ;$$

(2.5)

Where the number $\alpha$ is called the index of $\rho$.

Definition 2.3. The conventional Fourier-Stieltjes transform of a complex valued smooth function $f(t, x)$ is defined by the convergent integral.
\[ F(s, y) = FS\{ f(t, x) \} = \int_0^\infty f(t, x)e^{-ist}(x + y)^{-p} dt dx \]  
(2.6)

**Definition 2.4. Test function space: The Space \( FS_\alpha \)**

A function defined on \( 0 < t < \infty, 0 < x < \infty \) is said to be member of \( FS_\alpha \) if \( \phi(t, x) \) is a smooth and for each non-negative \( l, q \).

\[ y_{k, p, l, q} \phi(t, x) = \sup_{t} \left| t^{k} (1 + x)^{p} D_{l}^{q} (xD_{x})^{q} \phi(t, x) \right| \]  
(2.7)

\[ \leq C_{l, q} A^{p} p^{p} \quad p = 1, 2, 3, \ldots \]

Where the constant \( A \) and \( C_{l, q} \) depend on the testing function \( \phi \).

The space \( FS_\alpha \) are equiparallel with their natural Hausdorff locally topology \( \tau_\alpha \). This topology is respectively generated by the total families of semi norms \( \{ y_{k, p, l, q} \} \) given by (2.7).

**Definition 2.5. Distributional Fourier-Stieltjes transform of generalized function in \( FS_\alpha^* \)**

Let \( FS_\alpha^* \) is the dual space \( FS_\alpha \). This space \( FS_\alpha^* \) consists of continuous linear function on \( FS_\alpha \).

Let \( \phi(t, x) \in FS_\alpha^* \), for some \( s > 0 \) and \( k > \text{Re} \, p \), then distributional Fourier-Stieltjes Transform

\[ F(s, y) = \int f(t, x) e^{-ist} (x + y)^{-p} dt \]  
(2.8)

Where for each fixed \( t (0 < t < \infty) \) and \( x (0 < x < \infty) \) the right side of above equation has same as an application of \( f(t, x) \in FS_\alpha^* \) to \( e^{-ist} (x + y)^{-p} \in FS_\alpha \).

### 3. Main Results

Mainly, the results of this section are from [1]

#### 3.1. Tauberian theorems for Fourier –Stieltjes transformation

For main results of this section, we need the following assertion from [5]

**THEOREM 3.1** Suppose that for some \( m > 0 \) and

\[ x \to \infty, \quad \int_0^\infty \frac{d\phi(\lambda)}{(\phi + x)^{m+1}} \sim \int_0^\infty \frac{d\varphi(\lambda)}{(\phi + x)^{m+1}} \]  
(3.1)

and the following conditions are satisfied.

1. Functions \( \phi \) and \( \varphi \) are defined for \( x > 0 \) and are nondecreasing.
2. \( \lim_{x \to \infty} \phi(x) = \infty \).
3. For any \( C > 1 \), there are constants \( \gamma \) and \( N, 0 < \gamma < m, N > 0 \), such that for any \( x > y > N \),

\[ \frac{\phi(x)}{\phi(y)} \leq C \left( \frac{x}{y} \right) \]

Then for \( \lambda \to \infty, \phi(\lambda) \sim \varphi(\lambda) \).

(This means \( |\phi(\lambda)/\varphi(\lambda) - 1| < \epsilon \) if \( \lambda > \lambda_0(\epsilon) \), \( \lambda \in B \), means \((\lambda_0, \infty) \setminus B = 0 \).)

Let us note the condition (3) is called as the Keldysh-type condition.

Now we are ready to prove first Tauberian result.
THEOREM 3.2. Suppose that \( s > 1 \), \( r + m - s > 0 \), \( f \in FS_{s}^{+} \) and \( F \) is a nondecreasing function. Moreover, let

\[
\Gamma(r + 1)(FS\{ f(t,x) \}) \sim \frac{\Gamma(s) L(s)}{\Gamma(r + 1)} x^{r+s}, x \to \infty, t \to \infty
\]  

(3.2)

Where \( L \) is slowly varying function at \( \infty \) which is defined in some interval \([A, \infty)\), such that \( x^{r+k} L(x) \) is a nondecreasing function. Then \( f \) has quasiasymptotic behavior at \( \infty \) related to \( k r + m \) \( L(x) \) with the limit \( C x^{r+s} \), where \( C \neq 0 \).

Proof. Let us put

\[
\phi(x) = \begin{cases} 
  x^{r+m-s} L(x), & x > A \\
  0, & x \leq A
\end{cases}
\]  

(3.3)

Then \( \phi \) has the quasiasymptotic behavior at \( \infty \) related to \( k r + m L(x) \) with the limit \( f_{r+m+s+1} \).

Hence

\[
\int_{0}^{\infty} \frac{d\phi(t)}{(x+t)^{r+m}} = (r + m) \int_{0}^{\infty} \frac{\phi(t) dt}{x^{r+m+1}} \sim \frac{(r + m) \Gamma(s)}{\Gamma(r + m + 1)} x^{r+s}, x \to \infty, t \to \infty.
\]  

(3.5)

Now, we show that the conditions of Theorem 3.1 hold for \( \phi \) and \( F \). In fact, we have only to show that for some \( \gamma \), \( 0 < \gamma < r + m - 1 \), and every \( C > 1 \), there exists \( N > 0 \), such that

\[
\frac{\phi(\lambda y)}{\phi(y)} < C \lambda^{\gamma}, \quad \text{for} \; \lambda > 1, \; y > N
\]  

(3.6)

Let \( \gamma = r + m - s + \varepsilon \), where we choose \( \varepsilon > 0 \) such that \( \gamma > 0 \) and \( \varepsilon < s - 1 \). After substituting \( \phi \) in (3.6), we obtain

\[
L(\lambda y) \leq C \lambda^{\varepsilon} L(y)
\]  

and this inequality is true if \( \lambda > 1 \) and \( y > N \), where \( N \) depends on \( C \).

From the assumption that \( f \in FS_{s}^{+} \) and from (3.5) we have

\[
\Gamma(r + 1)(FS\{ f(t,x) \}) = (r + 1) \int_{0}^{\infty} \frac{F(t)}{(x+t)^{r+m+1}} dt 
\]

\[
= (r + 1) \int_{0}^{\infty} \frac{dF(t)}{(x+t)^{r+m}} \sim \frac{\Gamma(s) L(x)}{\Gamma(r + 1)} x^{r+s}, x \to \infty, t \to \infty
\]  

(3.7)

This implies that

\[
\Gamma(r + m + 1)(FS\{ f(t,x) \}) \sim \Gamma(r + m + 1)(FS\{ \phi(t,x) \}, x \to \infty, t \to \infty
\]  

(3.8)

And by Theorem 3.1, it follows that \( F \sim \phi, x \to \infty \).
Thus we obtain

\[
F(x) \sim \frac{(x^{r+m-s}L(x))}{\Gamma(r+m-s+1)}, x \to \infty
\] (3.9)

Since \(r + m - s > 0\), it follows that \(f\) has the quasisymptotic behavior at \(\infty\) related to \(k^{r+m-s}L(k)\) with the same limit \(x^r + m-s\).

Since \(f = t^{-r}D^mF\), it easily follows that \(f\) has the quasisymptotic behavior at \(\infty\) related to \(k^{r-s}L(k)\) with limit \(C x^{r-s}\), where \(C\) is a suitable nonzero constant. This completes proof of the theorem.

4. Tauberian-type results related to the quasisymptotic behavior

For the quasisymptotic behavior of all original \(f\) and for ordinary asymptotic of the corresponding function \(\Gamma(r+1)(FS\{f\})\), we need the following theorem and [5, Lemmas 1, 2, and 3]

**THEOREM 4.1.** Let \(n, m, n, m \in \mathbb{N}\) be a matrix of complex numbers.

(i) If \(a_{n,m}\) converges uniformly in \(m \in \mathbb{N}\) to \(a_m\) as \(m \to \infty\) and \(\lim_{n \to \infty} a_{n,m}\) exists, then

\[
\lim\lim_{n \to \infty, m \to \infty} a_{n,m} = \lim_{m \to \infty} a_{n,m} = \lim_{n \to \infty} a_{n,m} \tag{4.1}
\]

(ii) If \(\lim_{n \to \infty} a_{n,m}\) exists for every \(n \in \mathbb{N}\), \(\lim_{m \to \infty} a_{n,m}\) exists for every \(n \in \mathbb{N}\), \(\lim_{n, m \to \infty} a_{n,m}\) exists, then \(a_{n,m}\) converges uniformly in \(n \in \mathbb{N}\) as \(m \to \infty\).

**LEMMA 4.2.** Let \(r \in \mathbb{R} \setminus \{-\mathbb{N}\}, k \in \mathbb{N}_0\) be given and \(\gamma \in \mathbb{C}\), then for every \(n \in \mathbb{N}\),

\[
\sum_{i=1}^{n+1} (n+1)(-1)^i(2n+r+k+3)\cdots(2n+r+k-i)(2n+r+k+3-i)\cdots(r+k+\gamma+i+2) + (2n+r+k+\gamma+3)\cdots(r+k+\gamma+2) = (-1)^n \gamma(n-\gamma)\cdots(n-\gamma)(r+k+\gamma+2)_{(n+1)}.
\] (4.2)

**LEMMA 4.3.** Suppose that \(f \in S'\) and that \(f\) has the quasisymptotic behavior at \(\infty\) related to \(k^\nu L(k)\), where \(\nu < r\). Then there exists \(k \in \mathbb{N}_0\), \(k + \nu > 0\), and a continuous function \(F\) , \(\sup F \in [0, \infty)\), such that \(f = t^{-r}D^kF\) and \(F_1(x) = \int_0^x F(t)dt\), \(x \in \mathbb{R}\), \((\infty) (r+k+2)(T_{r+k+2}f_i(x) - F_i(x))/((1+x)^{r+k} L(x))\) converges uniformly to zero in \((0, \infty)\).

**LEMMA 4.4.** Let \(f \in FS_{\alpha}^+\) and \(\Gamma(r+1)(FS\{f\})(t, x) - x^{-\nu}t^\nu L(x), t \to \infty, x \to \infty, \nu > -1\).

Then \(\Gamma(r+1)(FS\{f\})_+(t, x)\) has the quasisymptotic behavior at \(\infty\) related to \(k^\nu L(k)\).

Proof. We have (\(\phi \in S\)),

\[
\frac{1}{k^\nu L(k)} \langle \Gamma(r+1)(FS\{f\})_+(kx), \phi(x) \rangle = \frac{1}{k^{\nu+1} L(k)} \langle \Gamma(r+1)(FS\{f\})_+(x), \phi\left(\frac{x}{k}\right) \rangle
\]

\[
= \frac{1}{k^{\nu+1} L(k)} \int_0^\infty \Gamma(r+1)(FS\{f\})(x) \times \phi\left(\frac{x}{k}\right) - \phi(0) - \cdots - \left(\frac{x}{k}\right)^{(l-1)}(0) \frac{d}{dx}
\]
Since the first part on the right -side of (4.3) converges to zero as $k \to \infty$, we have to prove that
\[
\frac{1}{k^r L(k)} \int_{\frac{x}{k}}^{x} \Gamma(r+1)(FS\{f\})(kx)\phi(x)dx \to \int_{0}^{\infty} x^r L(x)\phi(x)dx \text{ as } x \to \infty.
\]

Let us recall that $\Gamma(r+1)(FS\{f\})(t, x) \sim x^r L(x), x \to \infty$.

This implies that for a given $\varepsilon > 0$, there exists $x_0 > \varepsilon$, such that
\[
\left|\Gamma(r+1)(FS\{f\})(x) - x^r L(x)\right| \leq \varepsilon x^r L(x), \quad x \geq x_0 > 1
\]

We use the following decomposition:
\[
\frac{1}{k^r L(k)} \int_{\frac{x}{k}}^{x} \Gamma(r+1)(FS\{f\})(kx)\phi(x)dx
\]
\[
= \frac{1}{k^r L(k)} \left[ \int_{\frac{x_0}{k}}^{x_0} \Gamma(r+1)(FS\{f\})(kx)\phi(x)dx + \int_{x_0}^{\infty} \Gamma(r+1)(FS\{f\})(kx)\phi(x)dx \right] S
\]
\[
= \frac{1}{k^r L(k)} \int_{0}^{x_0} \Gamma(r+1)(FS\{f\})(kx)\phi(x)dx + \int_{x_0}^{\infty} \Gamma(r+1)(FS\{f\})(kx)\phi(x)dx
\]

The first member on the right-hand side of (4.5) tends to zero, when $k \to \infty$, because
\[
\frac{1}{k^r L(k)} \int_{\frac{x_0}{k}}^{x_0} \Gamma(r+1)(FS\{f\})(kx)\phi(x)dx \leq \frac{M}{k^r L(k)} \max_{x \leq k} |\phi(x)| \frac{x_0^r - 1}{k}
\]

Where $M = \max \{|\Gamma(r+1)(FS\{f\})(x)| : 1 \leq x \leq x_0\}$. Also one can prove easily that or a given $x_0 > 1$,
\[
\frac{1}{L(k)} \int_{0}^{x_0} x^r L(x)\phi(x)dx \to 0 \quad \text{as } k \to \infty
\]

Now by (4.4),(4.6),(4.7), we have
\[
\left| \frac{1}{k^r L(k)} \int_{\frac{x_0}{k}}^{x_0} \Gamma(r+1)(FS\{f\})(kx)\phi(x)dx - \frac{1}{k^r L(k)} \int_{0}^{x_0} (kx)^r L(kx)\phi(x)dx \right|
\]
\[
\leq \frac{1}{k^r L(k)} \int_{\frac{x_0}{k}}^{x_0} \Gamma(r+1)(FS\{f\})(kx)\phi(x)dx + \frac{1}{k^r L(k)} \int_{0}^{x_0} (kx)^r L(kx)\phi(x)dx
\]
\[
+ \frac{1}{k^r L(k)} \int_{\frac{x_0}{k}}^{x_0} \Gamma(r+1)(FS\{f\})(kx) - (kx)^r L(kx)\phi(x)dx
\]
\[
(4.8)
\]
Now we complete the proof of the lemma.

Now we will assume that I satisfies the inequality \( L(mx)/L(m) \leq C(1 + x), x > 0, m > 0. \)

Now we are ready to prove the following.

**Theorem 4.5.** If \( f \in S' \) and \( f \) has the quasiasymptotic behavior at \( \infty \) related to \( k^v L(k), r - 1 < v < r, r \in \mathbb{R} \setminus \{ -N \} \), then double sequence

\[
\frac{\Gamma(r + k + 2)(FS_{k+1}F_1)_{+}(xm)}{m^{v+k+1}L(m)}, \quad \phi^{(k+1)}(x)
\]

Converges to uniformly in \( n \in \mathbb{N} \) as \( m \to \infty \), where \( k \in \mathbb{N}, k + v > 0 \), and \( F_1 \) are defined in lemma 4.4.

Proof. Let us put

\[
a_{n,m} = \frac{\Gamma(r + k + 2)(FS_{k+1}F_1)_{+}(mx)\phi^{(k+1)}(x)}{m^{v+k+1}L(m)}, m, n \in \mathbb{N}, \phi \in S
\]

(4.9)

We have to prove that \( a_{n,m} \) converges uniformly in \( n \in \mathbb{N} \) as \( m \to \infty \).

First, we will prove that the conditions of theorem 4.1 (i) are satisfied. Then, from Theorem 4.1(ii), the aeration of this theorem will follow.

\[
a_{n,m} \to a_m = \frac{f(mx)}{m^vL(m)}, n \to \infty, m \in \mathbb{N}
\]

(4.10)

Since

\[
a_{n,m} - a_m = (-1)^{k+1}(r + 1)\int_{0}^{\infty} \frac{\Gamma(r + k + 2)(FS_{k+1}F_1)(mx)\phi^{(k+1)}(x)}{m^{v+k+1}L(m)} dx
\]

(4.11)

\[
= (-1)^{k+1}(r + 1)\int_{0}^{\infty} \left( \frac{\Gamma(r + k + 2)(FS_{k+1}F_1)(mx) - F_1(mx)(mx)}{mx(1 + mx)^{v+k}} \times \frac{(1 + mx)^v L(mx)}{m^{v+k+1}L(m)} \phi^{(k+1)}(x) \right) dx
\]

(4.12)

From Lemma 4.4 and the fact that

\[
\frac{(mx)(1 + mx)^v L(mx)}{m^{v+k+1}L(m)} \leq 2C(1 + x) \frac{mx(1 + (mx)^{v+k})}{m^{v+k+1}} \leq 2C(1 + x)(x^{v+k+1}), x \geq 0,
\]

(4.13)

We obtain that \( a_{n,m} - a_m \to \infty \) uniformly in \( n \in \mathbb{N} \) as \( n \to \infty \).

We have \( \Gamma(r + 1)(FS\{ f \})(x) \sim (\Gamma(r - v)/\Gamma(r + 1))x^{v-r+2} \) \( L(x), x \to \infty \). Since \( -1 < v - r \), we have by Lemma 4.4 that \( \Gamma(r + 1)(FS\{ f \})_{+} \) has the quasiasymptotic behavior at \( \infty \) related to \( k^{v-r} L(k) \) with the limit \( (\Gamma(r - v)/\Gamma(r + 1))x^{v-r} \)
By Leibniz formula we have

\[
D^{n+1} x^{2n+r+k+3} D^n (r + k + 2)(F_{k+1} F_1) (x)
\]

\[
= D^{n+1} \sum_{i=0}^{n+1} \binom{n+1}{i} (x^{2n+r+k+3})^i (r + k + 2)(F_{k+1} F_1) (x)^{n+1-i}
\]

\[
= \sum_{i=0}^{n+1} \binom{n+1}{i} (2n + r + k + 3) \cdots (2n + r + k + 4 - i)
\]

\[
\times (x^{2n+r+k+3-i}) (r + k + 3)(F_{k+1} F_1) (x)^{2n+2-i} + (x^{2n+r+k+3} \Gamma (r + k + 3)(F_{k+1} F_1 (x))^{2n+2})
\]

(4.15)

Let \( \gamma = u - r \). Then \( x^{2n+r+k+3} \Gamma (r + k + 2)(F_{k+1} F_1) (x) \) has quasiasymptotic behavior at \( \infty \) related to \( k^{r+k+\gamma+1} \).

Thus we obtain

\[
\lim_{n \to \infty} \frac{(-1)^n (1 - \gamma) \cdots (n - \gamma) (r + k + \gamma + 2)}{\Gamma (r + 1)(r + 1)} \Gamma (\gamma + \beta + k+1) x^{r+k+\gamma+1},
\]

(4.16)

To prove this last limit exists, we have to use Stirling formula.

\[
\Gamma (s+1) \sim 2\pi e^{-s} s^{(s+1)/2}, \ s \to \infty
\]

Thus for double sequence \( a_{n,m} \), Theorem 4.1(i) holds and Theorem 4.1(ii) implies assertion.

**THEOREM 4.6.** Let \( f \in FS^\alpha \) and let \( \Gamma (r + 1)(F_{k+1} F_1) (x) \) have quasiasymptotic at \( \infty \) related to \( k^{\alpha} L(k) \), \(-1 < \alpha < 0\). If for any \( \phi \in S \), the double sequence (4.9) converge uniformly in \( n \in \mathbb{N} \) as \( m \to \infty \), then \( f \) has the quasiasymptotic at \( \infty \) related to \( k^{\alpha} L(k) \).

Proof. If \( a_{n,m} \) is the double sequence defined in the proof of the Theorem 4.5, we have already by Theorem 4.1(i) the assertion.

**5. Conclusion**

This paper provides extension of distributional Fourier-Stieltjes transform, also Tauberian-type theorems for Fourier-Stieltjes transform are proved.

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