FUZZY GRACEFUL LABELING ON
GRAPHS RELATED TO CYCLE AND FAN

A. SOLAIRAJU\(^1\), and T. NARPPASALAI ARASU\(^2\)
\(^1\)Associate Professor of Mathematics, \(^2\)Assistant Professor in Mathematics
\(^1\)Jamal Mohamed College (Autonomous), Tirchy,
\(^2\)Chikkanna Government Arts College, Tirupur.

Abstract: Kishore and Sunitha [2014] identified chromatic number of few fuzzy graph. Denath [2013] investigated domination in interval-valued fuzzy graphs. Mathew and Mathew [2016] obtained some special degree-sequences in fuzzy graph. Tom and Sunitha [2016] destroyed boundary and interior nodes in a fuzzy graph using sum distance. Samanta and Pal [2015] stated on fuzzy planar graph. In this paper, fuzzy 10k-based-graceful labelings for the graphs \( F_n \oplus P_m, F_n \oplus K^*_{1,4} \) and \( C_n' \) are obtained.

Keywords: graceful labelling, fuzzy labelling, 10\(^1\)-based-graceful labeling.

I. Introduction:

II. Definitions and Preliminaries
Definition 2.1: Let \( U \) and \( V \) be two sets. \( \rho \) is a fuzzy relation from \( U \) into \( V \) if \( \rho \) is a fuzzy set of \( U \times V \).

Definition 2.2: A fuzzy graph \( G = (\sigma, \mu) \) is a pair of functions \( \sigma: V \rightarrow [0, 1] \) and \( \mu: V \times V \rightarrow [0, 1] \) such that \( \mu(uv) \leq \sigma(u) \land \sigma(v) \) for all \( u, v \in V \).

Definition 2.3: A labeling of a graph is an assignment of values to the vertices and edges of a graph.

Definition 2.4: A graceful labeling of a graph \( G \) with \( q \) edges is an injection \( f: V(G) \rightarrow \{0, 1, 2, ..., q\} \) such that each edge is assigned the distinct label \( |f(u) - f(v)| \).

Definition 2.5: A fuzzy graph \( G = (\sigma, \mu) \) has a fuzzy labeling if \( \sigma: V \rightarrow [0, 1] \) and \( \mu: V \times V \rightarrow [0, 1] \) are injective such that the membership value of edges and vertices are distinct, and \( \mu(uv) \leq \sigma(u) \land \sigma(v) \) for all \( u, v \in V \).

Definition 2.6: A fuzzy graph \( G = (\sigma, \mu) \) with \( p \) vertices and \( q \) edges. Choose a positive integer \( k \) with \( 0 < p + q < 10^k \). has a fuzzy \( 10^k \)-based-graceful labeling if \( \sigma: V \rightarrow [0, 1] \) and \( \mu: V \times V \rightarrow [0, 1] \) is bijective such that the membership value of edges and vertices are distinct, and \( \mu(uv) \leq \sigma(u) \land \sigma(v) \) for all \( u, v \in V \).

Definition 2.7: A fan \( F_n \) is obtained by \( F_n = K_1 + P_n \).

Definition 2.8: Choose a least positive integer \( k \) such that \( 0 < p + q < 10^k \).

III. Fuzzy 10\(^1\)-based graceful labeling:
Definition 3.1: The graph \( F_n \oplus P_m \) is a connected graph such that the vertex having the maximum degree in \( F_n \) is identified with any pendent vertex of \( P_m \).

Theorem 3.2: The graph \( F_n \oplus P_m \) is fuzzy 10\(^1\)-based-graceful.

Proof: Let \( v \) be the vertex having maximum degree of \( F_n \). One of the arbitrary labeling for the vertices of the graph \( F_n \oplus P_m \)

Consider a graph \( G = (\sigma, \mu) \) with \( p = (m+n+1) \), and \( q = (2n+m-1) \). Its vertex set is \( \{v, v_1, v_2, ..., v_m, v_{m+1}, v_{m+2}, ..., v_{m+n}\} \), and edge set is \( \{v_i v_{i+1}; i = 1 \ to \ m\} \cup \{v_m v_{m+1}; i = 1 \ to \ n\} \cup \{v_m v_{m+i+1}; i = 1 \ to \ n-1\} \).
Define $\sigma: V \rightarrow [0, 1]$ by the following rules:

$$\sigma(v_i) = \left\{ \begin{array}{ll}
\frac{i}{m} & \text{if } m \text{ is odd} \\
\frac{i}{m-1} & \text{if } m \text{ is even}
\end{array} \right.$$ 

for $i = 3, 5, \ldots, m$ (if $m$ is odd) / $m$ if $m$ is even;

$$\sigma(v_i) = \sigma(v_{i-2}) - \frac{1}{m-1}$$

for $i = 4, 6, \ldots, m-1$ (if $m$ is odd) / $m$ if $m$ is even;

$$\sigma(v) = \sigma(v_{m+1}) + \frac{1}{m-1}$$

if $m$ is odd;

$$\sigma(v') = \sigma(v_{m+1}) - \frac{1}{m}$$

if $m$ is even;

$$\sigma(v_{m+1}) = \sigma(v_{m-1}) + \frac{1}{m}$$

for $i = m+3, m+5, \ldots, m + n$ (if $m + n$ is odd) / $m + n - 1$ if $m + n$ is even.

Define $\mu: V \times V \rightarrow [0, 1] \text{ by } \mu(uv) = |\sigma(u) - \sigma(v)|$ for all $u,v \in V$. Then $\sigma$, and $\mu$ satisfy the fuzzy 10$^k$-based graceful labeling, and so $F_n \oplus P_m$ is fuzzy 10$^k$-based-graceful.

Example 3.3: Fuzzy 10$^k$-based-graceful labeling of $F_n \oplus P_m$ is shown in figure 1.

![Figure 1 - Fuzzy 10$^k$-based-graceful labeling of $F_n \oplus P_m$.](image)

Definition 3.4: $F_n \oplus K_{1,4}^+$ is a connected graph such that the vertex having maximum degree in $F_n$ is identified with the vertex having maximum degree in $K_{1,4}^+$.

Theorem 3.5: The graph $G = F_n \oplus K_{1,4}^+$ is fuzzy 10$^k$-based-graceful.

Proof: Let $V(G) = \{v_1, v_2, \ldots, v_{n+8}\}$ be the vertex set of $F_n$. Here $U = \{U_1, U_2\}$ be the partition of $K_{1,m}$ where $U_1 = \{v_1\}$ and $U_2 = \{v_{n+1}, v_{n+2}, v_{n+3}, v_{n+4}\}$ and $v_{n+5}$, $v_{n+6}$, $v_{n+7}$, $v_{n+8}$ be the pendant vertices joined with $v_{n+1}$, $v_{n+2}$, $v_{n+3}$, $v_{n+4}$ respectively.

Consider this graph $G = <\sigma, \mu>$ with $p = (n + 9)$ vertices, $q = (2n + 7)$ edges. Here $m = (2q + 1)$ such

Define $\sigma: V \rightarrow [0, 1]$ by the following rules:

$$\sigma(v) = \left\{ \begin{array}{ll}
\frac{i}{m} & \text{if } m \text{ is odd} \\
\frac{i}{m+1} & \text{if } m \text{ is even}
\end{array} \right.$$ 

for $i = 2, 4, \ldots, n, n+1$ (if $n$ is odd) / $n+1$ if $n$ is even;

$$\sigma(v_i) = \sigma(v_{i-2}) - \frac{1}{m}$$

for $i = 2, 4, \ldots, n, n+2$ (if $n$ is odd) / $n+2$ if $n$ is even;

$$\sigma(v_{n+1}) = \sigma(v) + \frac{1}{m+1}$$

$$\sigma(v_{n+3}) = \sigma(v) - \frac{1}{m+1}$$

$$\sigma(v_{n+4}) = \sigma(v)$$

Define $\mu: V \times V \rightarrow [0, 1] \text{ by } \mu(uv) = |\sigma(u) - \sigma(v)|$ for all $u,v \in V$.

Then $\sigma$, and $\mu$ satisfy the fuzzy 10$^k$-based graceful labeling, and so $F_n \oplus K_{1,4}^+$ is fuzzy 10$^k$-based-graceful.
Example 3.6: The fuzzy $10^k$-based graceful labeling of $F_6 \odot K_{1,4}$ is shown in figure 2.

Definition 3.7: A vertex switching $G_v$ of a graph $G$ is obtained by taking a vertex $v$ of $G$ removing all edges incidence to $v$ and adding edges joining $v$ to every vertex which are not adjacent to $v$ in $G$.

Theorem 3.8: The graph obtained by switching of a vertex in a cycle $C_n$ is fuzzy $10^k$-based-graceful.

Proof: Let $\{v_1, v_2, v_3, ..., v_n\}$ be the vertices of a cycle $C_n$. Assume that $C_n'$ is obtained by switching of a vertex in a cycle $C_n$. It is as follows.

Consider a cycle $C_n' = \langle \sigma, \mu \rangle$ with $p = n$ vertices, and $q = (2n-5)$ edges.

Define $\sigma: V \rightarrow [0, 1]$ by the following rules:

$\sigma(v_1) = (\frac{n-1}{n})$;

$\sigma(v_i) = \sigma(v_{i-2}) + (\frac{i-3}{n})$ for $i = 4, 6, ..., n-1$ (if $n$ is odd); $n-2$ if $n$ is even.;

$\sigma(v_2) = (\frac{n-1}{n})$;

$\sigma(v_i) = \sigma(v_{i-2}) - (\frac{i-3}{n})$ for $i = 5, 7, ..., n-2$ (if $n$ is odd); $n-1$ if $n$ is even.;

Define $\mu: V \times V \rightarrow [0, 1]$ by $\mu(uv) = |\sigma(u) - \sigma(v)|$ for all $u, v$ in $V$.

Then $\sigma$ and $\mu$ satisfy the fuzzy $10^k$-based graceful labeling, and so $C_n'$ is fuzzy $10^k$-based-graceful.

Example 3.9: The fuzzy $10^k$-based graceful labelling of $C_{10}'$ is shown in figure 3.
Conclusion: In this paper, fuzzy 10-based-graceful labelings of some graphs are obtained.

References: