TWO DIFFERENT CORRELATION COEFFICIENTS OF NEUTROSOPHIC FUZZY SETS

A. Solairaju, and M. Shajahan

1Associate Professor in Mathematics, 2Part-Time Research Scholar in Mathematics

1PG and Research Department of Mathematics, Jamal Mohamed College (A), Tiruchirappalli, India

Abstract: The correlation coefficient between fuzzy sets are analyzed in Chaudhuri and Bhattachary [2001]. Chiang and Lin [1999] studied correlation of intuitionistic fuzzy sets in Gerstenkorn & Manko [1991], Zeng & Li [2007], Hung & Wu [2002], and Mitchell [2004]. Park et.al. [2009] found correlation coefficient of interval-valued intuitionistic fuzzy sets and its application to multiple attribute group decision making problems. Robinson & Amirtharaj [2012] discussed vague correlation coefficient of interval vague sets. Wang & Li [1999] contributed correlation of information energy of interval valued fuzzy numbers. Wei, Wang, & Lin [2011] got application of correlation coefficient to interval vague sets and its application to interval valued intuitionistic fuzzy multiple attribute decision-making with incomplete weight information. In this paper, preliminaries of neutrosophic fuzzy sets and correlation coefficient of neutrosophic fuzzy sets are presented. Resistant correlation coefficient of neutrosophic fuzzy sets is also presented. Algorithms for two proposed model of MAGDM are explained. Different numerical illustration are given, dealing in three different ways of approaching an MAGDM problem when the weights are known and when the weights are unknown. Finally, some discussions and conclusions are made based on the models presented.

Keywords: Energy of neutrosophic fuzzy set, correlation coefficient of neutrosophic fuzzy sets, and hesitant-correlation coefficient of neutrosophic fuzzy sets;

I. INTRODUCTION: Group decision making problems can be robustly defined as decision situations where two or more decision makers or experts try to achieve a common solution to a decision problem, which consists of a set of possible solutions or alternatives. Experts must express their individual opinion on each of the alternatives. However, in many situations a problem arises when some experts consider that their individual opinions have not been sufficiently taken into account, and therefore they disagree with the solution achieved. This may lead to either a lack of implication in future group decision making problems or a behaviour against the solution obtained. For this reason, the need for making relevant decisions under consensus is becoming increasingly common in a variety of social situations.

Decision-making is the process of finding the best option from all of the feasible alternatives. Sometimes, decision-making problems considering several criteria are called multi-criteria decision-making (MCDM) problems. The MCDM problems may be divided into two kinds. One is the classical MCDM problems among which the ratings and the weights of criteria are measured in crisp numbers. Another is the fuzzy multiple criteria decision-making (FMCDM) problems, among which the ratings and the weights of criteria evaluated on imprecision and vagueness are usually expressed by linguistic terms, fuzzy numbers or intuition fuzzy numbers.

Atanassov [1986] introduced intuitionistic fuzzy sets. discussed correlation coefficient of interval-valued intuitionistic fuzzy sets was discussed in Burton and Burillo [1995], and Hong [1998]. Burton and Burillo [1996] found that vague sets are intuitionistic fuzzy sets. Partial correlation of fuzzy sets was explained in Chiang and Lin [2000], Murthy Pal & Majumder [1985] contributed correlation between two fuzzy membership functions. Szmidt & Kacprzyk [2000] explained distances between intuitionistic fuzzy sets.

II. BASIC DEFINITIONS AND OPERATIONS IN NEUTROSOPHIC FUZZY SETS

Definition 2.1: Let A = (T_A, I_A, F_A) and B = (T_B, I_B, F_B) be two neutrosophic fuzzy sets on an universe set X. The following definitions are defined.

(a). A \cup B = \{ x \in X \mid T_A(x) \vee T_B(x), I_A(x) \vee I_B(x), F_A(x) \wedge F_B(x) : x \in X \};
(b). A \cap B = \{ x \in X \mid T_A(x) \wedge T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x) : x \in X \};
(c). A' (x) = \{ x \in X \mid 1 - T_A(x), 1 - I_A(x) : x \in X \};
(d). (A \cdot B)(x) = \{ x \in X \mid T_A(x) \wedge T_B(x), I_A(x), I_B(x) : x \in X \};
(e). A \supseteq B if T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), and F_A(x) \geq F_B(x)

Result 2.2: The following properties are hold:
(i). 0_N \subseteq A \subseteq 1_N, \quad C(0_N) = 1, \quad C(1_N) = 0_N;
(ii). \quad A \subseteq B \iff C(B) \subseteq C(A); (iii). C(C(A)) = A;
(iv). \quad C(A \cup B) = C(A) \cap C(B); (v). C(A \cap B) = C(A) \cup C(B);
(vi). \quad A \subseteq B and E \subseteq D \iff A \subseteq B, D \subseteq E; (vii). A \subseteq B and E \subseteq D \iff A \subseteq B, D \subseteq E;
(viii). \quad A \subseteq B and E \subseteq D \iff A \subseteq B, E \subseteq D;
(ix). \quad A \subseteq E and B \subseteq D \iff A \subseteq B, E \subseteq D; (x). A \subseteq B and D \supseteq E , (y). A \subseteq B and E \subseteq D \iff A \subseteq B, E \subseteq D.

Properties 2.3: For A, B \in NFS(X), (i). 0 \leq (1/3n) C(A, B) \leq 1; (ii). C(A, B) = C(B, A);
(iii). C(A, B) = 1 if A = B.

© 2018 IJRAR September 2018, Volume 5, Issue 3 www.ijrar.org (E-ISSN 2348-1269, P-ISSN 2349-5138)

IJRAR1903764 | International Journal of Research and Analytical Reviews (IJRAR) www.ijrar.org | 909
III. CORRELATION COEFFICIENT OF NEUTROSOPHIC FUZZY SETS

Definition 3.1: A neutrosophic fuzzy set A on the universe of discourse X characterized by a truth membership function $T_A(x)$, an indeterminacy function $I_A(x)$ and a falsity membership function $F_A(x)$ is defined as $A = \{<x, T_A(x), I_A(x), F_A(x)> : x \in X\}$, where $T_A, I_A, F_A : X \rightarrow [0,1]$ and $0 \leq T_A(x) \leq 1; 0 \leq I_A(x) \leq 1; 0 \leq F_A(x) \leq 1$, for all $x \in X$.

For each IFS $A$ in $X$, $\pi_A(x) = 3 - T_A(x) - I_A(x) - F_A(x)$ is called as the neutrosophic index of $x$ in $A$. It is otherwise called as the hesitation degree of $x$ in $A$. It is obvious that $0 \leq \pi_A(x) \leq 1$ for each $x \in X$. For neutrosophic fuzzy sets $A, B$, define $A \subseteq B$ if $T_A(x) \leq T_B(x)$; $I_A(x) \leq I_B(x)$; $F_A(x) \leq F_B(x)$ for all $x \in X$.

Definition 3.2: Let $X = \{x_1, x_2, ..., x_6\}$ be a finite universal set and $A, B$ in NFS(X). Now, the method of calculating the covariance and the correlation coefficient between two NFSs is utilized proposed by us. The correlation coefficient of $A$ and $B$ is defined by the formula:

$$R_{NFS}(A, B) = \frac{C_{NFS}(A, B)}{\sqrt{E_{NFS}(A)E_{NFS}(B)}}$$

Theorem 3.3: For each $A, B$ in NFS $(X)$, the correlation coefficient satisfies:

(a). $R_{NFS}(A, B) = R_{NFS}(B, A)$; (b). $0 \leq R_{NFS}(A, B) \leq 1$. (c). $A = B$ iff $R_{NFS}(A, B) = 1$.

IV. PROPOSED MODEL FOR CORRELATION COEFFICIENT BETWEEN NEUTROSOPHIC FUZZY SETS

Algorithm 4.1: It has the following steps:

Step 1: Utilize the NFWOA operator to aggregate all individual neutrosophic fuzzy decision matrices $R^{(k)} = (r_{ij}^{(k)})_{m \times n}$ into a collective neutrosophic fuzzy decision matrix $R = (r_{ij})_{m \times n}$.

Step 2: For every $A \in NFS(X)$, $E_{NFS}(A) = (1/3n) \left[ \sum_{x \in X} T^A_i(x)^2 + (1 - I^A_i(x))^2 + (1 - F^A_i(x))^2 \right]$ defined to calculate the informational neutrosophic energy of $A$.

Step 3: For every $B \in NFS(X)$, $E_{NFS}(B) = (1/3n) \left[ \sum_{x \in X} T^B_i(x)^2 + (1 - I^B_i(x))^2 + (1 - F^B_i(x))^2 \right]$ defined to calculate the informational neutrosophic energy of $B$.

Step 4: The covariance $C_{NFS}(A, B) = \frac{1}{3n} \left[ \sum_{x \in X} (T^A_i(x)T^B_i(x) + (1 - I^A_i(x))(1 - I^B_i(x)) + (1 - F^A_i(x))(1 - F^B_i(x))) \right]$ for all $x$ in $X$ to calculate the covariance between the neutrosophic values $A$ and $B$.

Step 5: The correlation coefficient $R_{NFS}$ is calculated by equation $R_{NFS}(A, B) = \frac{C_{NFS}(A, B)}{\sqrt{E_{NFS}(A)E_{NFS}(B)}}$.

Example 4.2: Fort the variables $x_1, x_2, ..., x_6$, the corresponding neutrosophic fuzzy values of five items are as follows in a matrix form:

$$R^1 = \{<0.25, 0.54, 0.8>, <0.30, 0.40, 0.9>, <0.70, 0.35, 0.5>, <0.90, 0.20, 0.8>,$$
$$<0.60, 0.20, 0.5>, <0.60, 0.20, 0.3>, <0.20, 0.40, 0.9>, <0.60, 0.23, 0.7>,$$
$$<0.30, 0.45, 0.9>, <0.70, 0.10, 0.4>, <0.60, 0.5, 0.5>, <0.40, 0.20, 0.9>,$$
$$<0.45, 0.38, 0.27>, <0.37, 0.68, 0.16>, <0.60, 0.25, 0.3>, <0.90, 0.10, 0.4>\}.$$
Stage IV: Energy (R1) = 15.7692; Energy (R2) = 13.7151; Covariance (R1, R5) = 11.9244. Er(R1) . Er(R2) = 208.4983243 implies that $\sqrt{\text{Er}(R1)\text{Er}(R2)} = 14.43947105$. Correlation (R1, R3) = 0.2189. Hence R1 and R2 are highly correlated if R is considered.

V. PROPOSED MODEL FOR HESITATION-CORRELATION COEFFICIENT BETWEEN NEUTROSOHPIC FUZZY SETS:

Algorithm 5.1: It has the following steps.

Step 1: Utilize the NFOWA operator to aggregate all individual neutrosophic fuzzy decision matrices $R^{(x)} = (r^{(x)}_{ij})_{m \times n}$ (k = 1, 2, 3, 4) into a collective neutrosophic fuzzy decision matrix $R = (r_{ij})_{m \times n}$.

Step 2: $\pi(x)$ is the hesitation degree of a neutrosophic fuzzy set making truth sense. It is the complement of all truth values in the given neutrosophic fuzzy set and defined as $\pi(x) = 1 - (1 - T_A(x) + I_A(x) + F_A(x)) / 3$.

Step 3: For every $A \in NFS(X)$, $Expfs(A) = [\sum_{i=1}^{m} [I_A^2(x) + (1 - I_A(x))^2 + (1 - F_A(x))^2 + \pi(x)^2]]$ defined to calculate the informational neutrosophic energy of A.

Step 4: For every $B \in NFS(X)$, $Expfs(B) = [\sum_{i=1}^{m} [T_B^2(x) + (1 - I_B(x))^2 + (1 - F_B(x))^2 + \pi(x)^2]]$ defined to calculate the informational neutrosophic energy of B.

Step 5: The hesitation-covariance $C_{NF}(A, B) = \frac{1}{m} \sum_{i=1}^{m} [I_A^2(x)T_B^2(x) + (1 - I_A(x))(1 - I_B(x))(1 - F_A(x))(1 - F_B(x))]$ for all x in X to calculate the covariance between the neutrosophic values A and B.

Step 6: The hesitation-correlation coefficient $R_{NH}^{(x)}$ is calculated by equation $R_{NH}^{(x)}(A, B) = \frac{C_{NF}(A, B)}{\sqrt{Expfs(A) \cdot Expfs(B)}}.$

Example 5.2: From the neutrosophic fuzzy sets in example 1 for sixteen variables of five items, the following are neutron fuzzy sets with hesitant degree for each variable as given below through calculation hesitant degree by the rule in step 2.

R1={(<0.25,0.54,0.80,0.30>,<0.30,0.40,0.9>,0.33>,<0.70,0.35,0.50,0.62>,<0.90,0.20,0.80,0.63>,<0.60,0.50,0.50,0.54>,<0.60,0.20,0.3,0.63>,<0.20,0.40,0.9,0.3>,<0.60,0.23,0.76,0.56,0.36>,<0.32,0.67,0.56,0.36>,<0.32,0.67,0.56,0.36>,<0.65,0.25,0.32,0.69>,<0.60,0.30,1.07,13>,<0.75,0.25,0.55,0.65>,<0.27,0.9,0.81,0.19>,<0.31,0.4,0.6,0.44>,<0.75,0.65,0.55,0.52>,<0.30,0.7,0.9,0.23>}).

R2={(<0.32,0.47,0.60,0.42>,<0.90,0.10,0.30,0.83>,<0.60,0.40,0.50,0.57>,<0.30,0.50,0.70,0.37>,<0.12,0.32,0.52,0.43>,<0.17,0.81,0.9,0.15>,<0.50,0.30,1.07,1.7>,<0.45,0.65,0.27,0.51>,<0.50,0.6,0.23,0.56>,<0.56,0.52,0.23,0.54>,<0.30,0.60,1.05,53>,<0.57,0.52,0.55,0.5>,<0.54,0.83,0.72,0.33>,<0.73,0.36,0.51,0.42>,<0.50,0.52,0.40,0.53>,<0.65,0.40,0.20,0.67>}).

R3={(<0.70,0.30,1.07,1.77>,<0.50,0.40,0.40,0.57>,<0.20,0.10,0.65,0.5>,<0.70,0.9,0.6,0.4>,<0.30,0.56,0.70,0.34>,<0.57,0.24,0.10,74>,<0.23,0.76,0.65,0.27>,<0.53,0.50,0.27,0.54>,<0.32,0.32,0.60,0.47>,<0.56,0.52,0.32,0.57>,<0.10,0.30,9,0.3>,<0.57,0.52,0.55,0.5>,<0.72,0.50,0.18,0.68>,<0.13,0.50,0.4,0.38>,<0.55,0.55,0.78,0.40>,<0.70,0.10,0.60,0.57>}).

R4={(<0.52,0.45,0.10,0.66>,<0.57,0.37,0.10,0.7>,<0.76,0.65,0.23,0.63>,<0.57,0.52,0.55,0.5>,<0.30,0.6,0.70,0.33>,<0.70,0.40,1.07,13>,<0.30,0.70,0.60,0.3>,<0.05,0.40,0.60,0.5>,<0.20,0.30,0.20,0.57>,<0.60,0.20,0.50,0.63>,<0.10,0.60,0.65,0.28>,<0.30,0.90,0.70,23>,<0.27,0.50,0.81,0.32>,<0.75,0.25,0.32,0.73>,<0.32,0.67,0.56,0.36>,<0.35,0.56,0.72,0.39>}).

Stage I: Energy (R1) = 15.7692 + 4.3465 = 20.1157; Energy (R2) = 13.7316 + 4.301 = 18.0326; Covariance (R1, R2) = 12.3302 + 4.0376 = 16.3678. Er(R1) . Er(R2) = 362.7383718 implies that $\sqrt{\text{Er}(R1)\text{Er}(R2)} = 19.04569169$. Hesitant correlation (R1, R2) = 0.8589396459.

Stage II: Energy (R1) = 15.7692 + 4.3465 = 20.1157; Energy (R2) = 15.2124 + 4.4358 = 19.6482; Covariance (R1, R3) = 11.9506 + 3.8527 = 15.8033. Er(R1) . Er(R2) = 395.2372967 implies that $\sqrt{\text{Er}(R1)\text{Er}(R2)} = 19.88057587$. Hesitant correlation (R1, R3) = 0.794911581.

Stage III: Energy (R1) = 15.7692 + 4.3465 = 20.1157; Energy (R2) = 14.8494 + 4.457 = 19.3064; Covariance (R1, R4) = 11.8622 + 4.8368 = 16.6999. Er(R1) . Er(R2) = 388.3617505 implies that $\sqrt{\text{Er}(R1)\text{Er}(R2)} = 19.70689601$. Hesitant correlation (R1, R4) = 0.847368352.

Stage IV: Energy (R1) = 15.7692 + 4.3465 = 20.1157; Energy (R2) = 13.7151 + 4.3244 = 18.0395;
Covariance $(R_1, R_5) = 11.9244 + 4.7491 = 16.6735$.

$\text{Er}(R^1) \cdot \text{Er}(R^5) = 362.8771702 \text{ implies that } \sqrt{\text{Er}(R^1) \cdot \text{Er}(R^5)} = 19.04933516.$

Hesitant correlation $(R_1, R_5) = 0.87527989$.

Hence $R_1$ and $R_5$ are highly correlated under hesitant nature if $R_1$ is considered.

VI. Conclusion: A new approach for multiple attribute group decision making (MAGDM) problems where the attribute weights and the expert weights are real numbers and the attribute values take the form of neutrosophic fuzzy sets. Since families of ordered weighted averaging (OWA) operators are available in the literature, and only a few available for neutrosophic fuzzy sets, the vague ordered weighted averaging (VOWA) operator and the induced neutrosophic ordered weighted averaging (INOVA) operator are introduced in this paper and utilized for aggregating the neutrosophic information. The correlation coefficient for neutrosophic fuzzy sets is used for ranking the alternatives and a new MAGDM model is developed based on the INOWA operator and the neutrosophic fuzzy weighted averaging (NFWA) operator. In addition to the proposed model, two different models are proposed based on Linguistic Quantifiers for the situation when the expert weights are completely unknown. An illustrative example is given and a comparison is made between the models to demonstrate the applicability of the proposed approach of MAGDM.

References: