T-Anti Q Fuzzy subsemiring

B. Anitha
PG and Research Department of Mathematics,
Government Arts College, C.Mutlur, Chidambaram

Abstract: In this paper the concept of t- Anti Q Fuzzy subsemiring(normal subsemiring) and t- Anti Q Fuzzy ideals are introduced and some of its properties are discussed. The homomorphic behavior of t- anti Q Fuzzy subsemiring and t- anti Q Fuzzy ideals and inverse images are studied.

Keywords: Q Fuzzy set (QFS), Anti Q Fuzzy subsemiring (AQFSSR), Anti Q Fuzzy normal subsemiring (AQFNSSR), t-Anti Q fuzzy set (t-AQFS), t- Anti Q Fuzzy subsemiring (t-AQFSSR), t-Anti Q Fuzzy normal subsemiring (t-AQFNSSR).

1 Introduction

Using the notationof a fuzzy subset introduced by Zadeh [10], Liu [3] defined fuzzy set and fuzzy ideals of a ring. The notion of fuzzy subgroup was introduced by Rosenfeld [4] in his pioneering paper. Kim [2] studied intuitionistic Q-fuzzy semiprime ideals in semigroups. A Solairaju andR. Nagarajan [9] have introduced and defined a new algebraic structurecalled Q-fuzzy subgroups. S. Hemalatha, et.al. [1] introduced the concept ofQ-fuzzy subring of a ring and established some results. The notion of t-intuitionistic fuzzy subgroups, t-intuitionistic fuzzy quotient group and t-intuitionistic fuzzy subring of a ring have already been introduced by Sharma [5,6,7]. He [8] also introduced the concept of t-anti fuzzy subring and ideals of a ring.In this paper theconcept of t-anti Q fuzzy subsemiring(normalsubsemiring) and t-anti Q fuzzyideals of a ring are introduced. Some of its properties are discussed.

2 Preliminaries

Definition 2.1: Let (R,+,.) be a semiring. Let (Q,.) be a non-empty set. A mapA : Rx Q $\rightarrow$ [0, 1]is said to be a Q-fuzzy subset(QFS) of R.

Definition 2.2. Let (R,+,.) be a ring and Q be a non-empty set. A Q -fuzzy subset A of R is saidto be aAnti Q fuzzy subsemiring (AQFSSR) of R if it satisfies the following conditions:

(i) $A(x + y, q) \leq \max\{A(x, q), A(y,q)\}$

(ii) $A(xy, q) \leq \max\{A(x, q), A(y,q)\}$

Definition 2.3. Let (R,+,.) be a ring and Q be a non-empty set. A anti Q -fuzzy subsemiring A of R is saidto be a Anti Q fuzzy normal subsemiring (AQFNSSR) of R if $A(xy, q) = A(yx, q)$

Definition 2.4. Let A be a fuzzy set of a ring R. Let t ϵ [0,1]. Then the fuzzy set $A_t$of R is called the t-anti fuzzy subset (t-AFS) of R w.r.t. fuzzy set A and is defined as $A_t(x) = \max\{A(x),1-t\}$, for all x ϵ R.

Definition 2.5. Let A be a fuzzy set of a ring R and t ϵ [0,1].Then A is called t- anti fuzzy subring (in short t-AFSR) of R if A_t is fuzzy subring of R i.e. if the following conditions hold.

1. $A_t(x-y) \le \max\{A_t(x), A_t(y)\}$

2. $A_t(xy) \le \max\{A_t(x), A_t(y)\}$, for all x,y ϵR.

Definition 2.6. Let A_t be a anti fuzzy subring of a ring R and t ϵ [0,1]. Then A is called t- anti fuzzy normal subring (in short t-AFNSR) of R if A_t is fuzzy normal subring of R i.e. if $A_t(xy) = A_t(yx)$.

Definition 2.7:

Let A be a QFS of a ring R. Let tϵ[0,1]. Then A is called a

(a) Anti Q Fuzzy Left Ideal (AQFLI) of R if

(i) $A(x+y,q) \le \max\{A(x,q), A(y,q)\}$

(ii) $A(xy,q) \le A(y,q)$, for all x,yϵR and qϵQ.

(b) Q Fuzzy Right Ideal (AQFRI) of R if

(i) $A(x+y,q) \le \max\{A(x,q), A(y,q)\}$

(ii) $A(xy,q) \le A(x,q)$, for all x,yϵR and qϵQ.

(c) Q Fuzzy Ideal (AQFI) of R if

(i) $A(x+y,q) \le \max\{A(x,q), A(y,q)\}$

(ii) $A(xy,q) \le \min\{A(x,q), A(y,q)\}$ for all x,yϵR and qϵQ.

Definition 2.8: *Let f be a mapping from AQFSSR X to AQFSSR Y. For any fuzzy set B in Y, we define a new fuzzy set denoted as $f^{-1}(B) = B(f(x),q)$ for all $x \in X, q \in Q$. For any fuzzy set A in X, we define $f(A)$ in Y by*

$$f(A)(y,q) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} A(x,q), & \text{if } f^{-1}(y) \neq \phi \\ 0, & otherwise \end{cases}$$

3. t-Anti QFuzzy Subsemiring

Definition 3.1. Let A be a Q-fuzzy set (QFS) of a ring R. Let t ϵ [0,1]. Then the fuzzy set A_t of R is called the t-Anti QFuzzy subset (t-AQFS) of R with respect to QFS A and is defined as $A_t(x,q)=\max\{A(x,q),1-t\}$ for all x ϵR and q ϵQ.

Result 3.2. Let A_t and B_t be two t-AQFS of a ring R.

Then $(A\cup B)_t= A_t \cup B_t$.

Proof. Let x ϵR, q ϵQ be any element, then

$(A\cup B)_t (x,q) = \max\{(A\cup B)(x, q),1-t\} = \max \{\max\{A(x, q), B(x, q)\}, 1-t\}$

$= \max\{\max \{A(x, q),1-t\}, \max\{B(x, q)\},1-t\}\}= \max\{A_t(x, q), B_t(x, q)\}$

$= (A_t \cup B_t) (x, q)$

Hence $(A\cup B)_t=A_t\cup B_t$

Result 3.3. Let f : X →Y be a mapping. Let A and B are two QFS of X and Y respectively, then

i. $f^{-1}(B_t) = (f^{-1}(B))_t$

ii. $f(A_t) = (f(A))_t$, for all t ϵ [0,1]

Proof.

i. $f^{-1}(B_t)(x, q) = B_t(f(x),q) = \max\{ B(f(x),q), 1-t\} = \max\{ f^{-1}(B)(x, q), 1-t\}$
$= (f^{-1}(B))_t(x, q)$

Hence $f^{-1}(B_t) = (f^{-1}(B))_t$

ii. $f(A_t)(y,q) = \inf\{ A_t(x,q): (f(x).q)=(y,q)\} = \inf\{\max\{A(x,q),1-t\}: (f(x),q)=(y,q)\}$
$= \max\{\inf\{A(x,q) :(f(x),q)=(y,q)\},1-t\} = \max\{f(A)(y,q),1-t\} = (f(A))_t(y,q)$

Hence $f(A_t) = (f(A))_t$, for all t ϵ[0,1].

Definition 3.4. Let (R,+,.) be a ring and Q be a non-empty set. A Q–fuzzy subset A of R is saidto be a t-Qanti fuzzy subsemiring (t-QAFSSR)of R if it satisfies the following conditions:

(i) $A_t(x + y, q) \le \max\{A_t(x, q), A_t(y,q)\}$

(ii) $A_t(xy, q) \le \max\{A_t(x, q), A_t(y,q)\}$, for all x,yϵR and qϵQ.

Proposition 3.5: If A is a QAFSSR of a ring R, then A is called t-QAFSSR of R.

Proof. Let x,yϵR and qϵQ.

Now $A_t(x + y, q) = \max\{A(x+y,q),1-t\} \le \max\{ \max\{A(x,q),A(y,q)\},1-t\}$

$\le \max\{\max\{A(x,q),1-t\},\max\{A(y,q),1-t\}\} \le \max\{A_t(x, q), A_t(y,q)\}$

Hence $A_t(x + y, q \le \max\{A_t(x, q), A_t(y,q)\}$

Also $A_t(xy, q) = \max\{A(xy,q),1-t\} \le \max\{\max\{A(x,q),A(y,q)\},1-t\}$

$\le \max\{\max\{A(x,q),1-t\},\max\{A(y,q),1-t\}\} \le \max\{A_t(x, q), A_t(y,q)\}$

Hence $A_t(xy, q) \le \max\{A_t(x, q), A_t(y,q)\}$

Thus A is t-QFSSR of R.

Remark 3.6: The converse of the above proposition 3.5need not be true as the following example shows.

Example 3.7: Consider the Ring $(Z_5, +_4, x_4)$, where $Z_5=\{0,1,2,3,4\}$ and Q={q}. Define the fuzzy set A of Z_4 by

$$A(x,q) = \begin{cases} 0.3 \text{ if } x = 0 \\ 0.4 \text{ if } x = 1,3 \\ 0.6 \text{ if } x = 2,4 \end{cases}$$

Clearly A is not AQFSSR of Z_5. However, if we take t=0.3, then $A_t(x,q) = 0.7$, for all xϵZ_5 and qϵQ. Now A is a 0.3-AQFSSR of Z_5.

Proposition 3.8: Let A be aAQFSSR of the ring R. Let $t \le 1-p$ where $p = \sup\{A(x,q)$: for all xϵR, qϵQ}. Then A is t-AQFSSR of R.

Proof. Since $t \le 1-p \Rightarrow p \le 1-t \Rightarrow \sup\{A(x,q)$: for all xϵR, qϵQ$\} \le 1-t \Rightarrow A(x,q) \ge 1-t$ for all xϵR, and qϵQ.Therefore $A_t(x + y, q) \le \max\{A_t(x, q), A_t(y,q)\}$ and $A_t(xy, q) \le \max\{A_t(x, q), A_t(y,q)\}$ hold. Hence A is t-AQFSSR of R.

Proposition 3.9: The union of two t-AQFSSR's of R is also t-AQFSSR of R.

Proof. Let A and B be two t-AQFSSR of R. Let x,yϵR and qϵQ.

Now $(A\cup B)_t(x + y, q) = (A_t\cup B_t)(x+y, q) = \max\{A_t(x+y, q), B_t(x+y, q)\} \leq \max\{\max\{A_t(x,q), A_t(y,q)\}, \max\{B_t(x,q), B_t(y,q)\}\} = \max\{\max\{A_t(x,q), B_t(x,q)\}, \max\{A_t(y,q), B_t(y,q)\}\} = \max\{(A\cup B)_t(x, q), (A\cup B)_t(y, q)$

Thus $(A\cup B)_t(x + y, q) \leq \max\{(A\cup B)_t(x, q), (A\cup B)_t(y, q)$.

Also $(A\cup B)_t(xy, q) = (A_t\cup B_t)(xy, q) = \max\{A_t(xy, q), B_t(xy, q)\} \leq \max\{\max\{A_t(x,q), A_t(y,q)\}, \max\{B_t(x,q), B_t(y,q)\}\} = \max\{\max\{A_t(x,q), B_t(x,q)\}, \max\{A_t(y,q), B_t(y,q)\}\} = \max\{(A\cup B)_t(x, q), (A\cup B)_t(y, q)\}$

Thus $(A\cup B)_t(xy, q) \leq \max\{(A\cup B)_t(x, q), (A\cup B)_t(y, q)\}$.

Hence $(A\cup B)$is t-AQFSSR of R.

Proposition 3.10: Let A be AQFNSSR of a ring R, then A is also t-AQFNSSR of R.

Proof. Let x,yϵR and qϵQ. Then by above proposition A is t-AQFSSR of R. Now $A_t(xy,q)=\max\{A(xy,q), 1-t\}=\max\{A(yx,q),1-t\}=A_t(yx,q)$. Hence A is t-AQFNSSR of R.

Definition 3.11:

Let A be a QFS of a ring R. Let tϵ[0,1]. Then A is called a

a) t-Anti Q Fuzzy Left Ideal (t-AQFLI) of R if

(i) $A_t(x+y,q)\leq\max\{A_t(x,q), A_t(y,q)\}$

(ii) $A_t(xy,q)\leq A_t(y,q)$, for all x,yϵR and qϵQ.

b) t-Anti Q Fuzzy Right Ideal (t-AQFRI) of R if

(i) $A_t(x+y,q)\leq\max\{A_t(x,q), A_t(y,q)\}$

(ii) $A_t(xy,q)\leq A_t(x,q)$, for all x,yϵR and qϵQ.

c) t- Anti Q Fuzzy Ideal (t-AQFI) of R if

(i) $A_t(x+y,q)\leq\max\{A_t(x,q), A_t(y,q)\}$

(ii) $A_t(xy,q)\leq \min\{A_t(x,q), A_t(y,q)\}$for all x,y$\epsilon$R and q$\epsilon$Q.

Proposition 3.12: If A is AQFLI of a ring R, then A is also t-AQFLI of R.

Proof. Clearly $A_t(x+y,q)\leq\max\{ A_t(x,q), A_t(y,q)\}$(From Proposition 3.5)

Now $A_t(xy,q)=\max\{A(xy,q),1-t\}\leq\max\{A(y,q),1-t\}=A_t(y,q)$.

Hence A is t-AQFLI of R.

Proposition 3.13: If A is AQFRI of a ring R, then A is also t-AQFRI of R.

Proof. Clearly $A_t(x+y,q)\leq\max\{ A_t(x,q), A_t(y,q)\}$(From Proposition 3.5)

Now $A_t(xy,q)=\max\{A(xy,q),1-t\}\leq\max\{A(x,q),1-t\}=A_t(x,q)$.

Hence A is t-AQFRI of R.

Proposition 3.14: If A is AQFI of a ring R, then A is also t-AQFI of R.

Proof.Follows from Proposition 3.12 and Proposition 3.13.

4. Homomorphism of t- Anti Q Fuzzy Subsemiring:

Theorem 4.1: Let f: $R_1 \rightarrow R_2$ be a ring homomorphism from the Ring R_1 into a Ring R_2. Let B be t-AQFSSR of R_2. Then $f^{-1}(B)$ is t-AQFSSR of R_1.

Proof. Let B be a t-AQFSSR of R_2. Let $x_1, x_2 \epsilon R_1$ and $q \epsilon Q$. Then

$f^{-1}(B)_t(x_1+x_2,q)=f^{-1}(B_t)(x_1+x_2,q) = B_t(f(x_1+x_2),q) = B_t((f(x_1)+f(x_2)),q)) \leq \max\{B_t(f(x_1),q),B_t(f(x_2),q)\} \leq \max\{f^{-1}(B_t)(x_1,q), f^{-1}(B_t)(x_2,q)\}$

Now $f^{-1}(B_t)(x_1x_2,q) = B_t(f(x_1x_2,q)) = B_t((f(x_1)f(x_2)),q)) \leq \max\{B_t(f(x_1),q),B_t(f(x_2),q))\}$

$\leq \max\{f^{-1}(B_t)(x_1,q), f^{-1}(B_t)(x_2,q)\} = \max\{f^{-1}(B)_t(x_1,q),f^{-1}(B)_t(x_2,q)\}$

Hence $f^{-1}(B)$ is t-QFSSR of R_1.

Theorem 4.2: Let f: $R_1 \rightarrow R_2$ be a ring homomorphism from the Ring R_1 into a Ring R_2. Let B be t-AQFNSSR of R_2. Then $f^{-1}(B)$ is t-AQFNSSR of R_1.

Proof. Let B be t-AQFNSSR of R_2. Let $x_1, x_2 \epsilon R_1$ and $q \epsilon Q$.

$f^{-1}(B)_t(x_1x_2,q) = f^{-1}(B_t)(x_1x_2,q) = B_t(f(x_1x_2),q) = B_t(f(x_1)f(x_2),q) = B_t(f(x_2)f(x_1),q)$

$= B_t(f(x_2,q)\ f(x_1,q)) = B_t(f(x_2x_1),q)) = f^{-1}(B_t)(x_2x_1,q) = f^{-1}(B)_t(x_2x_1,q)$

Hence $f^{-1}(B)$ is t-QFNSSR of R_1.

Theorem 4.3: Let f: $R_1 \rightarrow R_2$ be a ring homomorphism from the Ring R_1 into a Ring R_2. Let B be t-AQFLI of R_2. Then $f^{-1}(B)$ is t-AQFLI of R_1.

Proof. Let B be t-AQFLI of R_2. Let $x_1, x_2 \epsilon R_1$ and $q \epsilon Q$.

Clearly $f^{-1}(B_t)(x_1+x_2,q) \leq \max\{f^{-1}(B_t)(x_1,q), f^{-1}(B_t)(x_2,q)\}$ (from theorem 4.1)

Now $f^{-1}(B_t)(x_1x_2,q) = B_t(f(x_1x_2),q) = B_t(f(x_1)f(x_2),q) \leq B_t(f(x_2),q) \leq f^{-1}(B_t)(x_2,q)$

Thus $f^{-1}(B)$ is t-QFLI of R_1.

Theorem 4.4: Let f: $R_1 \rightarrow R_2$ be a ring homomorphism from the Ring R_1 into a Ring R_2. Let B be t-AQFRI of R_2. Then $f^{-1}(B)$ is t-AQFRI of R_1.

Proof. It is similar to Theorem 4.3.

Theorem 4.5: Let f: $R_1 \rightarrow R_2$ be a ring homomorphism from the Ring R_1 into a Ring R_2. Let B be t-AQFI of R_2. Then $f^{-1}(B)$ is t-AQFI of R_1.

Proof. It can be obtained from Theorem 4.3 and Theorem 4.4.

Theorem 4.6: Let f: $R_1 \rightarrow R_2$ be surjective ring homomorphism and A be t-AQFSSR of R_1. Then f(A) is t-AQFSSR of R_2.

Proof. Since A is t-AQFSSR of R_1. Let $y_1, y_2 \epsilon R_2$ and $q \epsilon Q$. Then there exist some $x_1, x_2 \epsilon R_1$ such that $f(x_1)=y_1$ and $f(x_2)=y_2$.

$f(A_t)(y_1+y_2, q)=(f(A))_t(y_1+y_2, q)=\max\{f(A)(f(x_1)+f(x_2),q)),1-t\}$

$=\max\{f(A)(f(x_1+x_2),q),1-t\}=\max\{A(x_1+x_2,q),1-t\}=A_t(x_1+x_2,q)$

$\leq \max\{A_t(x_1,q),A_t(x_2,q)\}$, for all $x_1, x_2 \epsilon R_1$ such that $f(x_1)=y_1$ and $f(x_2)=y_2$

$\leq \max\{\wedge\{A_t(x_1,q):f(x_1)=y_1\}, \wedge\{A_t(x_2,q):f(x_2)=y_2\}\}=\max\{f(A_t)(y_1,q),f(A_t)(y_2,q)\}$

Thus $f(A_t)(y_1+y_2, q) \leq \max\{f(A_t)(y_1, q), f(A_t)(y_2, q)\}$

Now $f(A_t)(y_1y_2, q)=(f(A)_t(y_1y_2, q)=\max\{f(A)(f(x1)f(x_2),q),1-t\}$

$=\max\{f(A)(f(x_1x_2), q),1-t\} \leq \max\{A(x_1x_2, q),1-t\}=A_t(x_1x_2, q)$

$\leq \max\{A_t(x_1, q),A_t(x_2, q)\}$, for all $x_1, x_2 \epsilon R_1$ such that $f(x_1)=y_1$ and $f(x_2)=y_2$

$\leq \max\{\wedge\{A_t(x_1, q):f(x_1)=y_1\}, \wedge\{A_t(x_2, q):f(x_2)=y_2\}\}=\max\{f(A_t)(y_1, q),f(A_t)(y_2, q)\}$

Thus $f(A^t)(y_1y_2, q) \leq \max\{f(A_t)(y_1,q),f(A_t)(y_2, q)\}$

Hence f (A) is t-AQFSSR of R_2.

Theorem 4.7: Let f: $R_1 \rightarrow R_2$ be surjective ring homomorphism and A be t-AQFNSSR of R_1. Then f (A) is t-AQFNSSR of R_2.

Proof. Since A is t-AQFNSSR of R_1. Let $y_1, y_2 \epsilon R_2$ and$q \epsilon Q$.Then there exist some$x_1,x_2 \epsilon R_1$ such that $f(x_1)=y_1$ and $f(x_2)=y_2$. Clearly f(A) is t-AQFSSR of R_2 (from Theorem 4.6)

Now $(f(A))_t(y_1y_2, q)=f(A_t)(f(x_1)f(x_2), q)= f(A_t)(f(x_1x_2), q)=\wedge\{A_t(x_1x_2, q):f(x_1x_2)=y_1y_2\}$

$= \wedge\{A_t(x_2x_1, q):f(x_1x_2)=y_1y_2\} =f(A_t)(f(x_2x_1), q)=f(A_t)(f(x_2)f(x_1), q)=(f(A))_t(y_2y_1, q)$

Hence f (A) is t-AQFNSSR of R_2.

Theorem 4.8: Let f: $R_1 \rightarrow R_2$ be surjective ring homomorphism and A be t-AQFLI of R_1. Then f (A) is t-AQFLI of R_2.

Proof. Since A is t-AQFLI of R_1. Let $y_1, y_2 \epsilon R_2$ and $q \epsilon Q$. Then there exist some $x_1, x_2 \epsilon R_1$ such that $f(x_1)=y_1$ and $f(x_2)=y_2$. Clearly $f(A_t)(y_1+y_2, q) \leq \max\{f(A_t)(y_1, q),f(A_t)(y_2, q)\}$ (from Theorem 4.6)

Now $(f(A))_t(y_1y_2, q) = \max\{f(A)(f(x1)f(x_2),q)), 1-t\}=\max\{f(A)(f(x_1x_2), q),1-t\}$

$=\max\{A(x_1x_2, q),1- t\}=A_t(x_1x_2, q) \leq A_t(x_2, q)=\max\{A(x_2, q),1-t\}=\max\{f(A)((f(x_2),q),1-t\}=\max\{f(A)((y_2,q), 1-t\}=f(A)_t(y_2, q)$.

Thus $(f(A))_t(y_1y_2, q) \leq (f(A))_t(y_2, q)$

Hence f (A) is t-AQFLI of R_2.

Theorem 4.9: Let f: $R_1 \rightarrow R_2$ be surjective ring homomorphism and A be t-AQFRI of R_1. Then f (A) is t-AQFRI of R_2.

Proof. It is similar to Theorem 4.8

Theorem 4.10: Let f: $R_1 \rightarrow R_2$ be surjective ring homomorphism and A be t-AQFI of R_1. Then f (A) is t-AQFI of R_2.

Proof. It can be obtained from Theorem 4.8 and Theorem 4.9.

Reference:

1. Hemalatha.S, Naganathan.S and Arjunan.K*, Some theorem on Q-fuzzy subring of a ring*, International Journal of Emerging Technology and Advanced Engineering, 2(12), 2012, pp:581-585.
2. Kim.K.H.,*On intutionistic Q- fuzzy semiprime ideals in semigroups*, Advances in Fuzzy Mathematics, 1(1), 2006, PP:15-21.
3. Liu.W, *Fuzzy Invariant Subgroups And Fuzzy Ideals*, Fuzzy Sets and Systems, 8, 1982, pp:133-139.
4. Rosenfeld.A, *Fuzzy Groups*, Journal of Math, Anal. And Appl., 35, 1971, pp:512-517.
5. Sharma P.K., *t-intuitionistic fuzzy subgroups*, Int. J. Fuzzy Mathematics and Systems, 2(3), 2012, pp:233-243.
6. Sharma P.K., *t-intuitionistic fuzzy quotient group*, Advances in Fuzzy Mathematics, 7(1), 2012, pp:1-9.
7. Sharma P.K., *t-intuitionistic fuzzy subring*, IJMS, 11(3-4), 2012, pp:265-275.
8. Sharma P.K., *t-anti fuzzy subrings and ideals*, Indian Journal of Mathematics Research, 3(1), 2015, pp:47-57.
9. Solairaju.A and Nagarajan.R, *A new structure and construction of Q- fuzzy groups*, Advances in Fuzzy Mathematics, 4(1), 2009, 23-29.
10. Zadeh L.A, *Fuzzy sets*, Information and control, 8, 1965, pp:338-353.