PENTAGONAL FUZZY SEMI NUMBER WITH A CASE STUDY IN ANESTHESIA

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Abstract: In this research work, we define pentagonal fuzzy semi numbers and a method for finding distance between pair of pentagonal fuzzy semi numbers. By using this distance, we find a consistent nearest approximation of pentagonal fuzzy semi numbers with given height for given fuzzy semi number. We apply this concept in anesthesia case study.

Key words: Fuzzy semi numbers, Pentagonal fuzzy semi numbers, Pentagonal Height source distance, anesthesia

I. Introduction

In major surgeries, anesthetized patients are the part of study tool and the hemodynamic variables are controlled and monitored by an anesthetist in controlling the depth of anesthesia for balanced anesthesia. Providing adequate pain relief, unawareness of surroundings with sleeping induction drugs and good muscle relaxation form the component of balanced anesthesia. Under inhalational and intravenous drug usage during surgery, there will be a varied hemodynamic response in heart rate, blood pressure and other vital functions in the bodily system. The depth of anesthesia is a major factor.

Many theories have been proposed to handle this situation. The best theory is fuzzy set theory. Using this theory we can find more appropriate result. Many researchers developed the concept of fuzzy set, fuzzy logic and fuzzy control theory in medicine. They have contributed many approximation methods. All the methods destroy much of the useful data associated with fuzzy numbers resulting in imprecision in increment of approximation. To overcome this situation, Yeganehmanesh, Sh., Amirfakhrian, M., Grzegorzewski, P [14] introduced trapezoidal Fuzzy Semi numbers and a distance on them with a case study in medicine. This research paper works on fuzzy set with arbitrary height in comparison with an anesthesia case study using pentagonal fuzzy semi number.

This paper is organized as follows. In section 2, we present the concept of fuzzy semi numbers and pentagonal fuzzy semi numbers. In section 3, we propose a formula for height source distance between two pentagonal fuzzy semi numbers. In section 4, we find method to approximate arbitrary Pentagonal fuzzy semi number using pentagonal height source distance. In section 5, we developed a constraint for the nearest approximation of pentagonal fuzzy number to be a Pentagonal fuzzy semi number. Section 6 is dealt with numerical examples that explain the concept. Section 7 explains a real life application of fuzzy semi number in terms of anesthesia case study. Section 8 concludes the research work.

II. Preliminaries

Definition 2.1

A function $s \in C[0,1]$ is called source function over all fuzzy numbers if it satisfy the following conditions

a) $s(\alpha) \geq 0,\ \alpha \in [0,1]$.
b) $S(0) = 0$
c) $S(1) = 1$
d) $\int_0^1 s(\alpha)\, d\alpha = 1$

Definition 2.2

The core of a fuzzy number $\tilde{a}$ is denoted as core($\tilde{a}$) and is defined as core($\tilde{a}$)=$\{x \mid \mu_{\tilde{a}}(x) = 1\}$.

Definition 2.3

Let s be a source function defined on F(R). For $\tilde{a} \in$ F(R), value and ambiguity associated with s are defined as $v(\tilde{a}) = \int_0^1 s(\alpha)\left[\overline{u}(\alpha) + \underline{u}(\alpha)\right] d\alpha$ and $A(\tilde{a}) = \int_0^1 s(\alpha)\left[\overline{u}(\alpha) - \underline{u}(\alpha)\right] d\alpha$.

Definition 2.4

A fuzzy set $\tilde{a}$ is a generalized fuzzy semi number if there exist a positive h $\in$ (0,1], such that
$$\mu_{\tilde{a}}(x)=\begin{cases} l_1(x), & a\le x\le b\\ l_2(x), & b\le x\le c\\ h, & x=c\\ r_1(x), & c\le x\le d\\ r_2(x), & d\le x\le e\end{cases}$$
where $l_1(x)$, $l_2(x)$ is non decreasing on [a,b] & [b,c] respectively and $r_1(x), r_2(x)$ is non increasing on [c,d] &[d,e] respectively. $l_2(c)=r_1(c)=h$.

We denote this fuzzy semi number by $\tilde{a}_h=(a_1,a_2,a_3,a_4,a_5;h)$. The set of all fuzzy semi numbers is denoted by FS(R) and the set of all fuzzy semi numbers with height h is denoted as F_h(R). $FS(R)=\bigcup_{h\in(0,1]} F_h(R)$.

F(R) includes fuzzy set with height h=1 and F_h(R) includes fuzzy set with arbitrary height h. If $l_1(x)$, $l_2(x)$, $r_1(x), r_2(x)$ are linear then $\mu_{\tilde{a}}(x)$ is a pentagonal fuzzy semi number denoted by $(a_1,a_2,a_3,a_4,a_5;h)$. Let PF_h(R) be the set of all pentagonal fuzzy semi numbers on R with height h. PF_h(R)= $\{(a_1,a_2,a_3,a_4,a_5;h): a_1\le a_2\le a_3\le a_4, 0<h\le 1\}$. The set of all pentagonal fuzzy semi numbers with arbitrary heights by PFS(R). $PFS(R)=\bigcup_{h\in(0,1]} PF_h(R)$.

Definition 2.5

The core of a fuzzy number $\tilde{a}_h$ with height h ϵ(0,1] denoted as H-core($\tilde{a}_h$) and is defined as H-core($\tilde{a}_h$)= $\{x \mid \mu_{\tilde{a}}(x)=h\}$.

Definition 2.6

A function $s_h \in C[0,h]$ is called source function over all fuzzy numbers if it satisfy the following conditions

a) $s_h(\alpha)\ge 0$, $\alpha \in [0,1]$.

b) $s_h(0) = 0$

c) $s_h(h) = h$

d) $\int_0^h s_h(\alpha)\, d\alpha=\frac{h^2}{2}$.

Definition 2.7

Let s_h be a source function defined over [0,h]. For a fuzzy semi number $\tilde{a}_h$ ϵ F_h(R), Pentagonal H value and H-ambiguity associated with s_h are defined as

$$PHV=\int_0^h s_h(\alpha)[P_l(\alpha)+Q_l(\alpha)]d\alpha+s_h(\alpha)[P_u(\alpha)+Q_u(\alpha)]d\alpha \text{ and -----------(1)}$$

$$PHA=\int_0^h s_h(\alpha)[P_l(\alpha)+Q_l(\alpha)]d\alpha-s_h(\alpha)[P_u(\alpha)+Q_u(\alpha)]d\alpha \text{ ----------------(2)}$$

Where $P_l(\alpha)=a_1+\frac{\alpha(a_2-a_1)}{h}$, $P_u(\alpha)=a_5-\frac{\alpha(a_5-a_4)}{h}$, $Q_l(\alpha)=a_2+\alpha\left(\frac{a_3-a_2}{h}\right)$ and

$$Q_u(\alpha)=a_4-\frac{\alpha(a_4-a_3)}{h} \text{ .--------------------------(3)}$$

Properties 2.8

For $\tilde{a}_h$ ϵ F_h(R), and k ϵ R ,PHV and PHA satisfies the following

(i) PHV($\tilde{a}_h$ +k)=PHV($\tilde{a}_h$)+ h^2k.

(ii) PHA($\tilde{a}_h$ +k)=PHA($\tilde{a}_h$)

(iii) PHV($\tilde{a}_h \pm \tilde{b}_h$)=PHV($\tilde{a}_h$) ± PHV($\tilde{b}_h$)

(iv) PHA($\tilde{a}_h \pm \tilde{b}_h$)=PHA($\tilde{a}_h$) ± PHA($\tilde{b}_h$)

Definition 2.9

Crisp semi number a_h^0 is a fuzzy singleton set with height h then $l_2(\alpha) = r_2(\alpha) = a$ for α ϵ [0,h] and denote a_h^0=(a;h). The additive identity on $F_h(R)$ is $0_h^0 = (0; h)$.

Definition 2.10

Let S be a source function then the source number $I_{s,h}$ is defined on s as $I_{s,h} = \int_0^h s(\alpha)\,\alpha\, d\alpha$.-----------(4)

Definition 2.11

If s_1 and s_2 are equivalent source function defined over $[0,h_1]$ and $[0,h_2]$ respectively then $I_{s_2,h_2} = \left(\frac{h_2}{h_1}\right)^3 I_{s_1,h_1}$.

III. Height Source Distance between Pentagonal Fuzzy semi numbers

Definition 3.1

For $\tilde{a}_{h_a}, \tilde{b}_{h_b}$ ϵ PFS(R) with height h_a and h_b respectively then the height source distance between pentagonal fuzzy semi numbers(PHSD) is defined as

PHSD($\tilde{a}_{h_a}, \tilde{b}_{h_b}$)=

$$\begin{cases} \frac{1}{2}\left\{ \begin{aligned} & \left| h_a PHV(\tilde{a}_{h_a}) - h_b PHV(\tilde{b}_{h_b}) \right| + \left| h_a PHA(\tilde{a}_{h_a}) - h_b PHA(\tilde{b}_{h_b}) \right| + \\ & d_H\left(h_a^2 [\tilde{a}]^{h_a}, h_b^2 [\tilde{b}]^{h_b} \right) \end{aligned} \right\} & if\ \tilde{a} \neq 0^\circ_{h_a}\ or\ \tilde{b} \neq 0^\circ_{h_b} \\ |h_a - h_b| & if\ \tilde{a} = 0^\circ_{h_a}\ or\ \tilde{b} = 0^\circ_{h_b} \end{cases}$$ ---(5) where

$d_H\left(h_a^2 [\tilde{a}]^{h_a}, h_b^2 [\tilde{b}]^{h_b} \right)$=max $\left\{ \left| h_a^2 a_{2a_a} - h_b^2 a_{2b_b} \right|, \left| h_a^2 a_{3a_a} - h_b^2 a_{3b_b} \right|, \left| h_a^2 a_{4a_a} - h_b^2 a_{4b_b} \right| \right\}$-----(6), Hausdorff meter.

Properties 3.2

The following properties are satisfied by height source distance between pentagonal fuzzy semi numbers for all $\tilde{a}_{h_a}$, $\tilde{b}_{h_b}$ and $\tilde{c}_{h_a}$ ϵ PFS(R) and kϵR.

1. PHSD($\tilde{a}_{h_a}, \tilde{b}_{h_b}$) =0.
2. PHSD($\tilde{a}_{h_a}, \tilde{b}_{h_b}$)=PHSD($\tilde{b}_{h_b}, \tilde{a}_{h_a}$)
3. PHSD($\tilde{a}_{h_a}, \tilde{c}_{h_c}$)≤ PHSD($\tilde{a}_{h_a}, \tilde{b}_{h_b}$) + PHSD($\tilde{b}_{h_b}, \tilde{c}_{h_a}$)
4. PHSD($k\tilde{a}_{h_a}, k\tilde{b}_{h_b}$)= |k| PHSD($\tilde{a}_{h_a}, \tilde{b}_{h_b}$) .
5. PHSD($\tilde{a}_h + k\ ,\ \tilde{b}_h + k) = PHSD(\tilde{a}_h, \tilde{b}_h)$. $\forall \tilde{a}_h, \tilde{b}_h \in PF_h(R), k \in R.$
6. PHSD($\tilde{a}_h + \tilde{b}_h\ ,\ \tilde{a}'_h + \tilde{b}'_h) = PHSD(\tilde{a}_h, \tilde{a}'_h) + PHSD(\tilde{b}_h, \tilde{b}'_h)\ \forall \tilde{a}_h, \tilde{b}_h, \tilde{a}'_h, \tilde{b}'_h \in PF_h(R)$.

Proof is straightforward.

IV. The nearest approximation of pentagonal fuzzy semi number

Using PHSD, we find nearest approximation of arbitrary pentagonal fuzzy semi number in this section.

Definition 4.1

Let $\tilde{a}_h$ be an arbitrary fuzzy semi number. For h* $\in (0,1]$, $\tilde{a}_h{}^*$ is the consistent nearest approximation to $\tilde{a}_h$ out of PFS(R) , if and only if PHSD $\left(\tilde{a}_h, \tilde{a}^*_{h^*}\right) = 0$.

Theorem 4.2

Let $\tilde{a}_{h_a}, \tilde{b}_{h_b} \in$ PF$_h$(R) , then PHSD($\tilde{a}_{h_a}, \tilde{b}_{h_b}$) =0, if and only if $\tilde{a}_{h_a} = \tilde{b}_{h_b}$.

Proof:

If $\tilde{a}_{h_a} = \tilde{b}_{h_b}$ then it follows that PHSD($\tilde{a}_{h_a}, \tilde{b}_{h_b}$) =0.

Conversely assume that PHSD($\tilde{a}_{h_a}, \tilde{b}_{h_b}$) =0, then the following holds

(i) $d_H\left(h_a^2[\tilde{a}]^{h_a}, h_b^2\left[\tilde{b}\right]^{h_b} \right)$ =0.

(ii) $\left|h_a PHV\left(\tilde{a}_{h_a}\right) - h_b PHV\left(\tilde{b}_{h_b}\right)\right|$ =0.

(iii) $\left|h_a PHA\left(\tilde{a}_{h_a}\right) - h_b PHA\left(\tilde{b}_{h_b}\right)\right|$ =0.

from (i), we have max $\left\{\left|h_a^2 a_{2a} - h_b^2 a_{2b}\right|, \left|h_a^2 a_{3a} - h_b^2 a_{3b}\right|, \left|h_a^2 a_{4a} - h_b^2 a_{4b}\right|\right\}$=0 which implies $\left|h_a^2 a_{2a} - h_b^2 a_{2b}\right|$ =0 , $\left|h_a^2 a_{3a} - h_b^2 a_{3b}\right|$ =0 , $\left|h_a^2 a_{4a} - h_b^2 a_{4b}\right|$ =0.

Therefore we get, $a_{2b} = \dfrac{h_a^2}{h_b^2} a_{2a}$, $a_{3b} = \dfrac{h_a^2}{h_b^2} a_{3a}$ and $a_{4b} = \dfrac{h_a^2}{h_b^2} a_{4a}$ ------------------(7)

From (ii) and (iii) we have,

$$h_a PHV(\tilde{a}_{h_a}) + h_a PHA(\tilde{a}_{h_a}) \quad = h_b PHV(\tilde{a}_{h_b}) + h_b PHA(\tilde{a}_{h_b}) \text{-----------------(8)}$$

Using (1) ,(2) and (3) we get,

$$2\, h_a \int_0^{h_a} s_{h_a}(\alpha)\, \left[P_l(\alpha) + Q_l(\alpha)\right] d\alpha = 2\, h_b \int_0^{h_b} s_{h_b}(\alpha)\, \left[P_l(\alpha) + Q_l(\alpha)\right] d\alpha \text{ ---------(9)}$$

$$2\, h_a \int_0^{h_a} \alpha \left[a_{1a} + \alpha \frac{(a_{2a} - a_{1a})}{h_a} + a_{2a} + \alpha \frac{(a_{3a} - a_{2a})}{h_a} \right] d\alpha =$$

$$2 h_b \int_0^{h_b} \alpha \left[a_{1b} + \alpha \frac{(a_{2b} - a_{1b})}{h_b} + a_{2a} + \alpha \frac{(a_{3b} - a_{2b})}{h_b} \right] d\alpha .$$

On integrating we get,

$$a_{1a}\left(h_a^3 - 2I_{s,h_a}\right) + a_{2a} h_a^3 + 2a_{3a} I_{s,h_a} = a_{1b}\left(h_b^3 - 2I_{s,h_b}\right) + a_{2b} h_b^3 + 2a_{3b} I_{s,h_b} \text{ ----(10)}$$

$$I_{s,h_a} = \frac{h_a^3}{3},\ I_{s,h_b} = \frac{h_b^3}{3}.$$

Substitute the values of a_{2b} and a_{3b} from (7) in (10) and after simplification we have,

$$\frac{a_{1a}\left(h_a^3 - 2I_{s,h_a}\right)}{\left(h_b^3 - 2I_{s,h_b}\right)} + \frac{a_{2a}h_a^2\left(h_a - h_b\right)}{\left(h_b^3 - 2I_{s,h_b}\right)} + \frac{2a_{3a}\left(I_{s,h_a}h_b^2 - h_a^2 I_{s,h_b}\right)}{\left(h_b^3 - 2I_{s,h_b}\right)h_b^2} = a_{1b} \text{ ----------(11)}$$

Again From (ii) and (iii) we have,

$$h_a PHV(\tilde{a}_{h_a}) - h_a PHA(\tilde{a}_{h_a}) \quad = h_b PHV(\tilde{a}_{h_b}) - h_b PHA(\tilde{a}_{h_b}) \text{-----------------(12)}$$

Using (1) ,(2) and (3) we get,

$$2\,h_a \int_0^{h_a} s_{h_a}(\alpha)\,\left[P_u(\alpha) + Q_u(\alpha)\right] d\alpha = 2\ h_b \int_0^{h_b} s_{h_b}(\alpha)\,\left[P_u(\alpha) + Q_u(\alpha)\right] d\alpha \text{ ---------(13)}$$

$$2\,h_a \int_0^{h_a} \alpha\left[a_{5a} - \alpha\frac{(a_{5a} - a_{4a})}{h_a} + a_{4a} - \alpha\frac{(a_{4a} - a_{3a})}{h_a}\right] d\alpha =$$

$$2\,h_b \int_0^{h_b} \alpha\left[a_{5b} - \alpha\frac{(a_{5b} - a_{4b})}{h_b} + a_{4b} - \alpha\frac{(a_{4b} - a_{3b})}{h_b}\right] d\alpha \text{ ----------------(14)}$$

After integrating (14) we get,

$$a_{5a}\left(h_a^3 - 2I_{s,h_a}\right) + a_{4a}h_a^3 + 2a_{3a}I_{s,h_a} = a_{5b}\left(h_b^3 - 2I_{s,h_b}\right) + a_{4b}h_b^3 + 2a_{3b}I_{s,h_b} \text{ -----(15)}$$

$$I_{s,h_a} = \frac{h_a^3}{3},\ I_{s,h_b} = \frac{h_b^3}{3}.$$

Substitute the values of a_{4b} and a_{3b} from (7) in (15) and after simplification we have,

$$\frac{a_{5a}\left(h_a^3 - 2I_{s,h_a}\right)}{\left(h_b^3 - 2I_{s,h_b}\right)} + \frac{a_{4a}h_a^2\left(h_a - h_b\right)}{\left(h_b^3 - 2I_{s,h_b}\right)} + \frac{2a_{3a}\left(I_{s,h_a}h_b^2 - h_a^2 I_{s,h_b}\right)}{\left(h_b^3 - 2I_{s,h_b}\right)h_b^2} = a_{5b} \text{ -----------(16)}$$

We know that $h_a = h_b$ which implies that $a_{5a} = a_{5b}$ and $a_{1a} = a_{1b}$.
Hence $\tilde{a}_{h_a} = \tilde{b}_{h_b}$.

Corollary 4.3

Let $\tilde{a}_{h_a} \in PTF_{ha}(R)$, $\tilde{b}_{h_b} \in PTF_{hb}(R)$ and s_{h_a} be equivalent to s_{h_b}. If PHSD($\tilde{a}_{h_a}, \tilde{b}_{h_b}$) =0, then

$$\frac{a_{1a}\left(h_a^3\right)}{\left(h_b^3\right)} + \frac{a_{2a}h_a^2\left(h_a - h_b\right)}{\left(h_b^3 - 2I_{s,h_b}\right)} + \frac{2a_{3a}\left(h_b^2 - h_a^2\right)I_{s,h_a}}{\left(h_b^3 - 2I_{s,h_b}\right)h_b^2} = a_{1b} \text{ ---------------(17)}$$

$$a_{2b} = \frac{h_a^2}{h_b^2}a_{2a}\ ,\ a_{3b} = \frac{h_a^2}{h_b^2}a_{3a}\ ,\ a_{4b} = \frac{h_a^2}{h_b^2}a_{4a} \text{ -----------------(18)}$$

$$\frac{a_{5a}\left(h_a^3\right)}{\left(h_b^3\right)}+\frac{a_{4a}h_a^2\left(h_a-h_b\right)}{\left(h_b^3-2I_{s,h_b}\right)}+\frac{2a_{3a}\left(h_b^2-h_a^2\right)I_{s,h_a}}{\left(h_b^3-2I_{s,h_b}\right)h_b^2}=a_{5b} \text{------------------(19)}$$

For any arbitrary Pentagonal fuzzy semi number $\tilde{a}_{h_a} \in$ PTF$_{ha}$(R) with height h_a , we can find consistent nearest approximation $\tilde{b}_{h_b}=\left(a_{1b},a_{2b},a_{3b},a_{4b},a_{5b};h_b\right)$ by using equations (7), (11) and (16) for an arbitrary height h_b.

$$\begin{cases} a_{1b}=\dfrac{a_{1a}\left(h_a^3-2I_{s,h_a}\right)}{\left(h_b^3-2I_{s,h_b}\right)}+\dfrac{a_{2a}h_a^2\left(h_a-h_b\right)}{\left(h_b^3-2I_{s,h_b}\right)}+\dfrac{2a_{3a}\left(I_{s,h_a}h_b^2-h_a^2I_{s,h_b}\right)}{\left(h_b^3-2I_{s,h_b}\right)h_b^2} \\ a_{2b}=\dfrac{h_a^2}{h_b^2}a_{2a} \\ a_{3b}=\dfrac{h_a^2}{h_b^2}a_{3a} \\ a_{4b}=\dfrac{h_a^2}{h_b^2}a_{4a} \\ a_{5b}=\dfrac{a_{5a}\left(h_a^3-2I_{s,h_a}\right)}{\left(h_b^3-2I_{s,h_b}\right)}+\dfrac{a_{4a}h_a^2\left(h_a-h_b\right)}{\left(h_b^3-2I_{s,h_b}\right)}+\dfrac{2a_{3a}\left(I_{s,h_a}h_b^2-h_a^2I_{s,h_b}\right)}{\left(h_b^3-2I_{s,h_b}\right)h_b^2} \end{cases} \text{----------(20)}$$

The nearest approximations of known pentagonal fuzzy semi number with height h_a need not be pentagonal fuzzy semi number with some height h_b. This fact can be illustrated by the following example.

Example 4.4:

Let $\tilde{a}$ =(1,2,3,4,5;1/2) and s(α)=α, then the consistent nearest approximations of $\tilde{a}$ with height 1/3 is given by $\tilde{b}=\left(\frac{135}{8},\frac{9}{2},\frac{27}{4},\frac{9}{1},\frac{729}{108};\frac{1}{3}\right)$.

Here $a_{1b} \geq a_{2b}$, hence $\tilde{b}$ is not pentagonal fuzzy semi number.

To solve this, we are going to find constraints on the height of consistent nearest approximation in the next section.

V. A constraint on the nearest approximation height of pentagonal fuzzy semi number

Let $\tilde{a}=\left(a_{1a},a_{2a},a_{3a},a_{4a},a_{5a};h_a\right)$ be an arbitrary pentagonal fuzzy semi number then the constraint for the nearest approximation through PHSD with height h_b where $\tilde{b}=\left(a_{1b},a_{2b},a_{3b},a_{4b},a_{5b};h_b\right)$ can be derived from following results.

Result 5.1

(i) If $h_b \geq h_a$, the H-core of the consistent nearest approximated pentagonal fuzzy semi number $\tilde{b}_{h_b}$ tends to the left side.

(ii) If $h_a \geq h_b$, the H-core of the consistent nearest approximated pentagonal fuzzy semi number $\tilde{b}_{h_b}$ tends to the right side.

Proof:

(i) Suppose $h_b \geq h_a$ then from (20) we yield, $a_{2b} \leq a_{2a}$, $a_{3b} \leq a_{3a}$ and $a_{4b} \leq a_{4a}$. Hence the approximation pentagonal fuzzy semi number $\tilde{b}_{h_b}$ tends to the left side.

(ii) Suppose $h_a \geq h_b$ then from (20) we get, $a_{2b} \geq a_{2a}$, $a_{3b} \geq a_{3a}$ and $a_{4b} \geq a_{4a}$. therefore the approximation pentagonal fuzzy semi number $\tilde{b}_{h_b}$ tends to the right side.

Result 5.2

Let s be a source function for an arbitrary known pentagonal fuzzy semi number $\tilde{a} = (a_{1a}, a_{2a}, a_{3a}, a_{4a}, a_{5a}; h_a)$ with height h_a then the consistent nearest approximation through PHSD is a pentagonal fuzzy semi number with height h_b if and only if h_b satisfies

(i) If $h_b \geq h_a$, then $h_b \leq \dfrac{3a_{4a}h_a^3 + 3a_{5a}I_a + 6a_{3a}I_a}{2(2a_{4a} - a_{3a})h_a^2}$, $I_a = I_{s,h_a}$

(ii) If $h_a \geq h_b$, then $h_b \geq \dfrac{3a_{1a}I_a + 3a_{2a}h_a^3 + 2a_{3a}I_a}{2h_a^2(5a_{2a} + a_{3a})}$, $I_a = I_{s,h_a}$

Proof:

Let $I_a = I_{s,h_a}$, $I_b = I_{s,h_b}$.-------(21)

Since $\tilde{a}_{h_a}$ is pentagonal fuzzy semi number $a_{1a} \leq a_{2a} \leq a_{3a} \leq a_{4a} \leq a_{5a}$ -----(22)

The consistent nearest approximation of $\tilde{a}_{h_a}$ is $\tilde{b}_{h_b} = (a_{1b}, a_{2b}, a_{3b}, a_{4b}, a_{5b}; h_b)$, a pentagonal fuzzy semi number if $a_{1b} \leq a_{2b} \leq a_{3b} \leq a_{4b} \leq a_{5b}$.

It is clear that $a_{2b} \leq a_{3b} \leq a_{4b}$ from (20)

If $h_a \geq h_b$ then to prove $\tilde{b}_{h_b}$ is pentagonal fuzzy semi number, it is enough to prove that $a_{1b} \leq a_{2b}$.

Let us consider $a_{2b} - a_{1b} = \dfrac{h_a^2}{h_b^2}a_{2a} - \dfrac{a_{1a}\left(h_a^3 - 2I_{s,h_a}\right)}{\left(h_b^3 - 2I_{s,h_b}\right)} + \dfrac{a_{2a}h_a^2(h_a - h_b)}{\left(h_b^3 - 2I_{s,h_b}\right)} + \dfrac{2a_{3a}\left(I_{s,h_a}h_b^2 - h_a^2 I_{s,h_b}\right)}{\left(h_b^3 - 2I_{s,h_b}\right)h_b^2}$

Apply definition 2.11 , (21),(22) we get,

$$= \frac{h_a^2 a_{2a}\left(h_b^3 - 2\dfrac{h_b^3}{h_a^3}I_a\right) - a_{1a}h_b^2\left(h_a^3 - 2I_a\right) + a_{2a}h_a^2(h_a - h_b)h_b^2 + 2a_{3a}\left(I_a h_b^2 - \dfrac{h_a^2 h_b^2}{h_a^3}I_a\right)}{h_b^2\left(h_b^3 - 2\dfrac{h_b^3}{h_a^3}I_a\right)}$$

$$= \frac{h_a^2 a_{2a}\left(h_a^3 - 2h_b^3 I_a\right) - a_{1a}\left(h_a^3 - 2I_a\right) + a_{2a}h_a^2(h_a - h_b) + 2a_{3a}\left(I_a - \dfrac{h_a^2}{h_a^3}I_a\right)}{\left(h_b^3 - 2\dfrac{h_b^3}{h_a^3}I_a\right)}$$

Simplification gives $h_b \geq \dfrac{3a_{1a}I_a + 3a_{2a}h_a^3 + 2a_{3a}I_a}{2h_a^2(5a_{2a} + a_{3a})}$ -------(23)

Similarly, to prove (i) consider

$a_{5b} - a_{4b} = \dfrac{a_{5a}\left(h_a^3\right)}{\left(h_b^3\right)} + \dfrac{a_{4a}h_a^2(h_a - h_b)}{\left(h_b^3 - 2I_{s,h_b}\right)} + \dfrac{2a_{3a}\left(h_b^2 - h_a^2\right)I_{s,h_a}}{\left(h_b^3 - 2I_{s,h_b}\right)h_b^2} - \dfrac{h_a^2}{h_b^2}a_{4a}$

$$= \frac{-h_a^2 a_{4a}\left(h_b^3 - 2\dfrac{h_b^3}{h_a^3}I_a\right) - a_{5a}h_b^2\left(h_a^3 - 2I_a\right) + a_{4a}h_a^2(h_a - h_b)h_b^2 + 2a_{3a}\left(I_a h_b^2 - \dfrac{h_a^2 h_b^2}{h_a^3}I_a\right)}{h_b^2\left(h_b^3 - 2\dfrac{h_b^3}{h_a^3}I_a\right)}$$

$$= \frac{-h_a^2 a_{4a}\left(h_a^3 - 2h_b^3 I_a\right) - a_{5a}\left(h_a^3 - 2I_a\right) + a_{4a}h_a^2\left(h_a - h_b\right) + 2a_{3a}\left(I_a - \frac{h_a^2}{h_a^3} I_a\right)}{\left(h_b^3 - 2\frac{h_b^3}{h_a^3} I_a\right)}$$

Applying definition 2.11, (21) and (22) and after simplification we get

$$= \frac{h_a^3}{h_b^3}\left[2I_a h_b(a_{4a} + a_{3a}) + h_a^4(a_{5a} + a_{4a}) + 2h_a I_a(a_{3a} - a_{5a}) - 2a_{4a}h_a^3 h_b\right] \geq 0$$

Implies that

$$h_b \leq \frac{3a_{4a}h_a^3 + 3a_{5a}I_a + 6a_{3a}I_a}{2(2a_{4a} - a_{3a})h_a^2} \text{.----------------(24)}$$

VI. Numerical Examples

Example 6.1

Let $\tilde{a} = (1,2,3,4,5;0.5)$ and s(α) = α. The consistent nearest pentagonal fuzzy number of $\tilde{a}$ with height 0.47 is $\tilde{b}$ =(1.9,2.26,3.4,4.5,7.6).

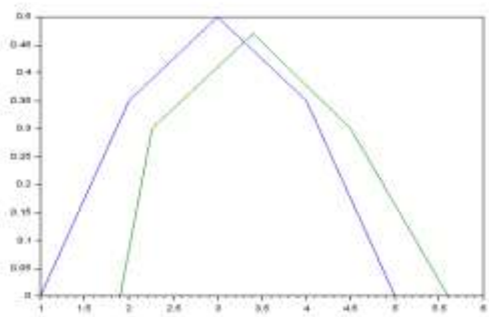

Figure 1

Example 6.2

Let $\tilde{a} = (1,2,3,4,5;0.5)$ and s(α) = α. The consistent nearest pentagonal fuzzy number of $\tilde{a}$ with height 0.44 is $\tilde{b}$ =(2.5,2.58,3.87,5.17,8.7).

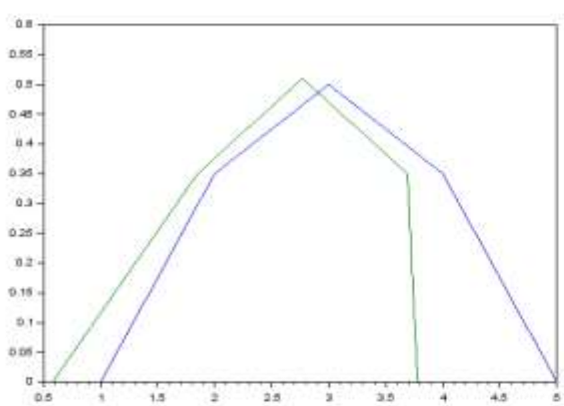

Figure 2

Example 6.3

Let $\tilde{a} = (1,2,3,4,5;0.5)$ and s(α) = α. The consistent nearest pentagonal fuzzy number of $\tilde{a}$ with height 0.51 is $\tilde{b}$ =(0.764,1.92,2.88,3.84,4.25).

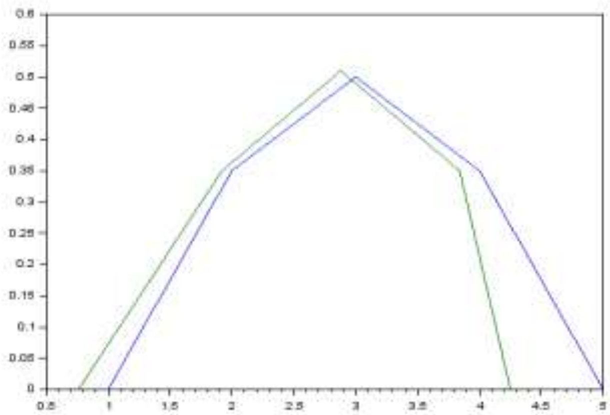

Figure 3

Example 6.4

Let $\tilde{a} = (1,2,3,4,5;0.5)$ and s(α) = α. The consistent nearest pentagonal fuzzy number of $\tilde{a}$ with height 0.52 is $\tilde{b}$ =(0.583,1.85,2.77,3.69,3.77).

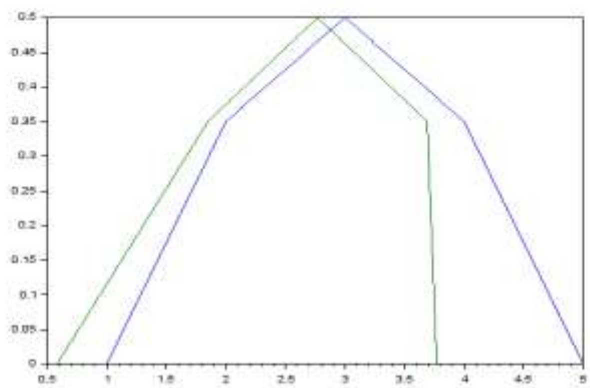

Figure 4

In all the above figures , given pentagonal fuzzy numbers are drawn in blue lines and consistent nearest approximation pentagonal fuzzy semi numbers are drawn in green lines. X axis represents given fuzzy number and Y axis represents their height.(arbitrarily chosen)

VII.Medical case Study

Anaesthetist plays a pivotal action in controlling the depth of anesthesia during surgery. Their main role is to reduce the morbidity and mortality in patients undergoing major surgeries. Hence anesthesia solve the problems through calculating some confounding factors . Cullen[9] in his research showed that there is a reasonable correlation between blood pressure and anesthetic dose in patients undergoing General anesthesia. In other words, gaining a depth of anesthesia is feasible through controlling the blood pressure related factors. It should fall within a predetermined range. Maintaining the depth of anesthesia is of high importance to both the anesthetist and surgeon.Sieber [13] and colleagues reported accurate control of mean alveolar concentration of isoflurane by a system that altered the gas flow rates.

Zbinden et al [17] employed a system that regulated the inspired isoflurane concentration in response to changes in blood pressure, alternating this with manual techniques. The fuzzy logic system commences with the skin incision of the patient till the end of the surgery.

There are two ways to anesthetize a patient, one is performed by intravenous injection of drugs or inhaling gases which is a mixture of Isoflurane or Sevoflurane in oxygen with nitrous oxide of appropriate mixtures. The concentration of anesthetic gases used in the process is determined and tuned regarding the type of surgery and the patient's clinical condition. Minimum alveolar concentration is the determinant of partial pressure of the anesthetic agents in the brain reflecting the depth of anesthesia. The anesthetist are the decision maker and controller for adequate depth of anesthesia.

In this research paper, a fuzzy logic controller is developed to measure the Mean Arterial Pressure. This simulates the relationship between the inflow concentration of Isoflurane or Sevoflurane and the blood pressure. The average of patient's blood pressure is given as fuzzy semi number. The blood pressure measurement is ambiguous because of some disturbances and hence the given data should be fuzzy set. There are different types of disturbances such as surgical disturbances and measurement noise that cause ambiguity in the measurement of blood pressure to increase. To overcome this, for a given fuzzy semi number we find the range of heights which indicates blood pressure variation. We then find a new fuzzy semi number with given height related to new blood pressure that helps the anesthetist to determine the dose of drugs through the anesthetic process.

Consider a pentagonal fuzzy semi number $\tilde{a} = (1,2,3,4,5;0.9)$ represents the uncertainity in each blood pressure measurement. The height of given fuzzy semi number is 0.9 is proportional to blood pressure and its support is [1,5] shows the anesthetic duration.

We can find the range of height association with the given fuzzy semi number using (23) and (24) as $0.31 \le h_b \le 1.5$

This range shows the change in blood pressure with the inflow concentration of Isoflurane or Sevoflurane by using defuzzification methods in medicine. If we want to change the blood pressure we approximate new fuzzy semi number with height 0.88 related to new blood pressure by using (20)

$\tilde{b} = (1.31, 2.09, 3.14, 4.18, 5.69)$

This pentagonal fuzzy semi number $\tilde{a}$ and consistent nearest approximation of pentagonal fuzzy semi number $\tilde{b}$ is depicted in picture as

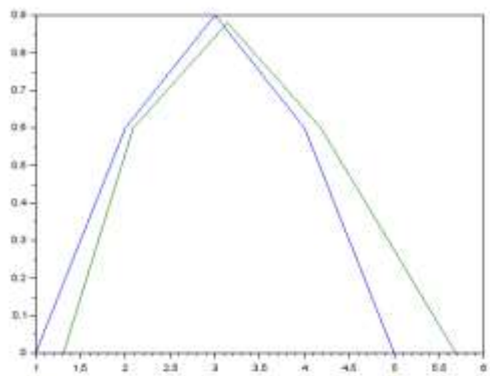

Figure 5

Using this consistent nearest pentagonal fuzzy semi number, the anesthetist will easily determine the next drug application for the patient.

VII.Conclusion

In this paper, we define pentagonal fuzzy semi number. We developed pentagonal height source distance to approximate a arbitrary pentagonal fuzzy semi number. Also we derived the range of height associated with given fuzzy semi number and using this height, a consistent nearest approximation of given fuzzy semi number is found. We presented enough examples to describe the concept. Also we presented a medical case study which improved anesthetic decision for prescribing dose of drugs and controlling the depth of anesthesia with our proposed method.

REFERENCES

[1] Abbasbandy, S., Asady, B.2004. The nearest trapezoidal fuzzy number to a fuzzy quantity. Appl. Math. Comput. 156, 381–386 .

[2] Abbasbandy, S., Amirfakhrian, M.2006. The nearest trapezoidal form of a generalized left right fuzzy number. J. Approx. Reason. 43, 166–178

[3] Abbasbandy, S., Hajjari, T.2009.A new approach for ranking of trapezoidal fuzzy numbers.Comput. Math. Appl. 57, 413–419 .

[4] Abbasbandy, S., Nuraei, R., Ghanbari, M.2009.Revision of sign distance method for ranking of fuzzy numbers. Iran. J. Fuzzy Syst. 10(4), 101– 117

[5] Amirfakhrian, M.2010. Properties of parametric form approximation operator of fuzzy numbers. Analele Stiintifice ale UniversitatiiOvidius Constanta 18, 23–34 .

[6] Chanas, S.2001.On the interval approximation of a fuzzy number. Fuzzy Sets Syst. 122, 353–356 .

[7] Cheng, C.H.1998.A new approach for ranking fuzzy numbers by distance method, Fuzzy Sets and Systems, 95 , 307–317.

[8] Coroianu, L., Gagolewskic, M., Grzegorzewski, P.2013. Nearest piecewise linear approximation of fuzzy numbers. Fuzzy Sets Syst. 233, 26–51

[9] Cullen, D.J., et al.1972. Clinical signs of anesthesia. Anesthesiology 36, 21–36 .

[10] Delgado, M., Vila, M.A., Voxman, W.1998. On a canonical representation of fuzzy numbers.Fuzzy Sets Syst. 93, 125–135.

[11] Grzegorzewski, P.2002. Nearest interval approximation of a fuzzy number. Fuzzy Sets Syst. 130, 321–330 .

[12] Li, G., Warner, M., Lang, B.H., Huang, L., Sun, L.S.2009.Epidemiology of anesthesia-related mortality in the United States, 1999–2005. Anesthesiology 110(4), 759–765 .

[13] Sieber TJ, Frei CW, Derightetti M, Feigenwinter P, Leibundgut D, Zbinden AM.2000. Model based automatic feedback control versus human control of end- tidal isoflurane concentration using low- flow anesthesia. British Journal of Anaesthesia;85, 818-25.

[14] Yeganehmanesh, Sh., Amirfakhrian, M., Grzegorzewski, P.2018. Fuzzy Semi numbers and a distance on them with a case study in medicine. Mathematical Sciences 12, 41-52.

[15] Ying, H., Sheppard, L.C.1994. Regulating mean arterial pressure in postsurgical cardiac patients. IEEE Eng. Med. Biol. Mag. 13(5), 671–677.

[16] Zbinden, A.M., Feigenwinter, P., Petersen-felix, S., Hacisalizade, S.1995 Arterial pressure control with isoflurane using fuzzy logic. Br. J. Anaesth. 74, 66–72 .

[17] Zimmermann, H.J.1991. Fuzzy Set Theory and its Applications, 2nd edn. Kluwer Academic, Boston .