MV- Optimality of Nearest Neighbour Balanced Block Designs using Autoregressive Moving Average Model (ARMA (1,1)) for Seven Treatments

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Abstract
Neighbour balanced block designs for observations correlated within a block have been investigated. MV - Optimality of block designs for three treatments in \( b = 3n \pm 1 \) blocks each of size three under the assumption that blocks behave independently was addressed by Nizam Uddin. Ruban Raja and Santharam were investigated MV - Optimality of Nearest Neighbour Balanced Block Designs (NNBD) using first order Correlated Models for Five treatments. In this paper, we have investigated MV - Optimality of Nearest Neighbour Balanced Block Designs using first order Autoregressive Moving Average Model (ARMA (1,1)) for Seven treatments.

Key words: Neighbour Balanced Design, MV-optimality, Optimal Design, Autoregressive Moving Average Model, Experimental design.

1. Introduction
Serology is the scientific study of serum and other bodily fluids. In practice, the term usually refers to the diagnostic identification of antibodies in the serum. Such antibodies are typically formed in response to an infection against other foreign proteins (in response, for example, to a mismatched blood transfusion), or to one's own proteins. Serological methods are diagnostic methods that are used to identify antibodies and antigens in patient’s sample which is serum and plasma. Many studies concerned with viral preparation require the arrangement of antigens in a place so that each antigen has two other antigens as its neighbours. In analysis of such experiment the classical design may not perform efficiently. Therefore Rees, D. H. (1967) introduced neighbour structure. The following is the experiment considered by Rees, D. H. (1967) Nearest Neighbour Balanced Block Design. If the observations are available are correlated, therefore the usual assumption like independence of observation in the analysis of classical comparative experiments may not be valid. Therefore, there is necessity for the use of Nearest Neighbour Balanced Block Design. In biometrical sciences we can cite many areas where this kind of correlated structure exists. Now consider the viral preparations. Let there be \( v \) kinds of antigens to be arranged on \( b \) plates, each containing \( k \) antigens. Each antigen appears \( r \) times (but not necessarily on \( r \) different plates) and is a neighbour of every other antigen exactly \( \lambda \) times. Rees used circular neighbouring block design and he was used incomplete neighbor design \((k < v)\) in his experiment. The parameters of the design are \( v = 9, b = 9, k = 4, r = 4, \lambda = 1 \) and the 9 plates are

\[ P_1 = (5, 6, 4, 1), P_2 = (6, 7, 5, 2), P_3 = (7, 8, 6, 3), P_4 = (8, 9, 7, 4), P_5 = (9, 1, 8, 5), P_6 = (1, 2, 9, 6), P_7 = (2, 3, 1, 7), P_8 = (3, 4, 2, 8), P_9 = (4, 5, 3, 9) \]

In the present paper, we have taken complete NNBD \((k = v)\) with the parametric structures, \( v = 7, b = 7, k = 7, r = 7, \lambda = 2 \) and investigated the optimality of NNBD (for \( \rho_1 = 0.1; \rho_1 = 0.2; \ldots; \rho_1 = 0.9 \) where \( \rho_1 \) is the correlation coefficients) when the errors behaving according to ARMA (1,1) model. Uddin, N., (2008) has constructed MV- Optimality of block design for 3 treatments in \( b = 3n \pm 1 \) blocks of each size 3 and under the assumption that the blocks behave independently but there is a correlation among the observations with the same block according to AR (1) Model.

2. Nearest Neighbour Balanced Block Design
Neighbour balanced block designs, where in the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbours, are used for modelling and controlling interference effects between neighbouring plots. Neighbor designs provide a tool for local control in biometrics, agriculture, horticulture and forestry. Neighbor effects (either natural or due to layout of plots) can deprive the results from its representativeness. Therefore, it is important to control it through design or analysis. Ahmed, R and Akhtar, M were addressed the Construction of Neighbor Balanced Block designs (2008). Design is a specification of the preparations made before the measurement e. g. allocation of treatments to experimental units and analysis is drawing conclusions about the treatments from the result of the experiment. Neighbor designs are used to study the neighbor effects among neighboring units. Neighbor effects can be caused by differences in height, root vigor, or germination date of plant in agriculture. Similarly, fertilizer, irrigation, or pesticide applied on one plot may cause neighbor effect to its adjacent plots.

3. Optimal Designs
Optimal designs may be optimal in many different ways, and what may be an optimal design according to one criterion may be suboptimal for other criteria. An optimality criterion provides a measure of the fit of the data to a given hypothesis, to aid in model selection. A model is designated as the "best" of the candidate models if it gives the best value of an objective function measuring the
degree of satisfaction of the criterion used to evaluate the alternative hypotheses. Competing criteria have led to a literal alphabet-soup collection of optimal design methodologies. Optimal designs offer three advantages over suboptimal experimental designs. Optimal designs can accommodate multiple types of factors, such as process, mixture and discrete factors. Designs can be optimized when the design space is constrained. In practical terms, optimal experiments can reduce the costs of experimentation.

4. MV – Optimality
A design \( d^* \in D \) is said to be MV-optimal if

\[
\max_{i \neq j} var_{d^*}(\hat{t}_i - \hat{t}_j) \leq \max_{i \neq j} var_d(\hat{t}_i - \hat{t}_j)
\]

where \( d \) is any other competing design in \( D \).

4.1 Construction of the Model
A block design \( d \) is defined here as an allocation of \( v \) treatments to \( bk \) experimental units which are arranged into \( b \) blocks each having \( k \) units. Such a design is said to be MV – optimal if it minimizes, with respect to all designs in a well-defined class of designs, the largest variance of the estimates of treatment differences. The MV-optimality problem is addressed here in the class \( D_b \) of all equireplicate connected design for \( v = 7, k = 7, b = 3n \pm 1, n \geq 1 \), assuming that the block observations within the same block are correlated.

We assume the following model,

\[
Y_d = 1_{3b\times t} + Z\beta + X_d\tau + \epsilon \quad \text{with} \quad \text{cov} \ \epsilon = \Sigma
\]

where, \( Y_d = \) block order \( 3b \times 1 \) column vector of observed response obtained from a design \( d \),
\( 1_{3b} = 3b \times 1 \) column vector of ones
\( \tau = 3 \times 1 \) vector of treatment effect
\( X_d = 3b \times 3 \) plot-treatment design matrix
\( \beta = 3 \times 1 \) vector of fixed block effects
\( Z = I_b \otimes 1 \) plot-block incident matrix.

If the errors within a block follow a first order autoregressive moving average model ARMA (1,1) with the parameter \( \rho \) (where \( \rho \) is the correlation between the observations in the adjacent plots) then,

\[
\Sigma = I_b \otimes \begin{bmatrix}
    r_0 & r_1 & r_2 & \ldots & r_{k-1} \\
    r_1 & r_0 & r_2 & \ldots & r_{k-2} \\
    r_2 & r_1 & r_0 & \ldots & r_{k-3} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    r_{k-1} & r_{k-2} & r_{k-3} & \ldots & r_0
\end{bmatrix}
\]

for all \( \rho \geq 0 \)

\[
r_0 = \frac{1 - 2\rho \rho_2 + \rho_2^2}{1 - \rho_1^2}
\]

\[
r_1 = \frac{\rho_1(1 + \rho_2^2) + \rho_2(1 + \rho_1^2)}{1 - \rho_1^2}
\]

\[
r_k = \rho_1 r_{k-1}, \text{for } k \geq 2
\]

4.2 Information Matrix
The least square information matrix,

\[
C_d = X_d'\Sigma^{-1}X_d - X_d'\Sigma^{-1}Z(Z'\Sigma^{-1}Z)^{-1}Z'\Sigma^{-1}X_d
\]  

(4.2.1)

The above matrix is utilized by several authors (e.g. Martin and Eccleston, 1991; Jin and Morgan, 2008; Gill and Shukla, 1985; Kunert, 1987; Santharam and Ponnumswamy, 1995, 1996, 1997; Uddin, 2008a, 2008b) in their investigation of various optimal and highly efficient design. Universal optimal designs which includes MV- Optimal designs are often characterized using two sufficient conditions of Kiefer’s (1975) Proposition 1*: find a design \( d' \) such that trace \((C_d')\) is maximized with respect to all designs in \( D_b \) and that \((C_d')\) is completely symmetric.
5. Variance of The Generalized Least Squares Estimates of Treatment Differences

Let $C_{dij}$ denote the $(i,j)^{th}$ element of $C_d$, for any $d \in D_b$, the following inequalities hold (Lee and Jacroux, 1987),

$$Var_d (\hat{t}_i - \hat{t}_j) \geq \frac{c_{dij}^2 + c_{dij} + 2c_{dij}}{c_{dij}}$$

5.1 Variance of the Generalized Least Squares Estimates of Seven treatments Differences

For an arbitrary design $d \in D$, if we let $C_{dij}$ denote the $(i,j)^{th}$ element of $C_d$, then the variance of the estimates of treatment differences (apart from the constant $\sigma^2$) may be expressed as follows:

$$Var_d (\hat{t}_1 - \hat{t}_2) = \frac{b_{11} + b_{22} + 2b_{12}}{det_d}$$

$$Var_d (\hat{t}_1 - \hat{t}_3) = \frac{b_{11} + b_{33} + 2b_{13}}{det_d}$$

$$Var_d (\hat{t}_1 - \hat{t}_4) = \frac{b_{11} + b_{44} + 2b_{14}}{det_d}$$

$$Var_d (\hat{t}_1 - \hat{t}_5) = \frac{b_{11} + b_{55} + 2b_{15}}{det_d}$$

$$Var_d (\hat{t}_1 - \hat{t}_6) = \frac{b_{11} + b_{66} + 2b_{16}}{det_d}$$

$$Var_d (\hat{t}_1 - \hat{t}_7) = \frac{b_{11}}{det_d}$$

$$Var_d (\hat{t}_2 - \hat{t}_3) = \frac{b_{22} + b_{33} + 2b_{23}}{det_d}$$

$$Var_d (\hat{t}_2 - \hat{t}_4) = \frac{b_{22} + b_{44} + 2b_{24}}{det_d}$$

$$Var_d (\hat{t}_2 - \hat{t}_5) = \frac{b_{22} + b_{55} + 2b_{25}}{det_d}$$

$$Var_d (\hat{t}_2 - \hat{t}_6) = \frac{b_{22} + b_{66} + 2b_{26}}{det_d}$$

$$Var_d (\hat{t}_2 - \hat{t}_7) = \frac{b_{22}}{det_d}$$

$$Var_d (\hat{t}_3 - \hat{t}_4) = \frac{b_{33} + b_{44} + 2b_{34}}{det_d}$$

$$Var_d (\hat{t}_3 - \hat{t}_5) = \frac{b_{33} + b_{55} + 2b_{35}}{det_d}$$

$$Var_d (\hat{t}_3 - \hat{t}_6) = \frac{b_{33} + b_{66} + 2b_{36}}{det_d}$$

$$Var_d (\hat{t}_3 - \hat{t}_7) = \frac{b_{33}}{det_d}$$

$$Var_d (\hat{t}_4 - \hat{t}_5) = \frac{b_{44} + b_{55} + 2b_{45}}{det_d}$$

$$Var_d (\hat{t}_4 - \hat{t}_6) = \frac{b_{44} + b_{66} + 2b_{46}}{det_d}$$

$$Var_d (\hat{t}_4 - \hat{t}_7) = \frac{b_{44}}{det_d}$$

$$Var_d (\hat{t}_5 - \hat{t}_6) = \frac{b_{55} + b_{66} + 2b_{56}}{det_d}$$

$$Var_d (\hat{t}_5 - \hat{t}_7) = \frac{b_{55}}{det_d}$$

$$Var_d (\hat{t}_6 - \hat{t}_7) = \frac{b_{66}}{det_d}$$

Generalized inverse of $C_d$ matrix ($C_d^{-1} = X'V^{-1}X - (X'V^{-1}Z)(Z'V^{-1}Z)^{-1}(Z'V^{-1}X)$) used in the calculation of the above variance is computed by replacing the leading 2x2 sub matrix by its inverse and replacing the elements in the last row and last column by zero (N. Uddin, 2008b). In this synopsis, we have analyzed the above variance is computed by replacing 3x3 sub matrix by its inverse and replacing the elements in the last row and last column by zero.
5.2 MV – Optimal Designs for Seven Treatments

A design $d^* \in D$ is said to be MV – optimal iff.

$$\max \{ \text{var}_d^* (\hat{\tau}_2 - \hat{\tau}_1), \text{var}_d^* (\hat{\tau}_1 - \hat{\tau}_3), \text{var}_d^* (\hat{\tau}_1 - \hat{\tau}_4), \text{var}_d^* (\hat{\tau}_1 - \hat{\tau}_5), \text{var}_d^* (\hat{\tau}_2 - \hat{\tau}_3), \text{var}_d^* (\hat{\tau}_2 - \hat{\tau}_4), \text{var}_d^* (\hat{\tau}_2 - \hat{\tau}_6), \text{var}_d^* (\hat{\tau}_3 - \hat{\tau}_7), \text{var}_d^* (\hat{\tau}_3 - \hat{\tau}_5), \text{var}_d^* (\hat{\tau}_3 - \hat{\tau}_6), \text{var}_d^* (\hat{\tau}_4 - \hat{\tau}_5), \text{var}_d^* (\hat{\tau}_4 - \hat{\tau}_6), \text{var}_d^* (\hat{\tau}_4 - \hat{\tau}_7), \text{var}_d^* (\hat{\tau}_5 - \hat{\tau}_6), \text{var}_d^* (\hat{\tau}_5 - \hat{\tau}_7), \text{var}_d^* (\hat{\tau}_6 - \hat{\tau}_7) \}$$

$$\leq \max \{ \text{var}_d^* (\hat{\tau}_1 - \hat{\tau}_2), \text{var}_d^* (\hat{\tau}_1 - \hat{\tau}_3), \text{var}_d^* (\hat{\tau}_1 - \hat{\tau}_4), \text{var}_d^* (\hat{\tau}_1 - \hat{\tau}_5), \text{var}_d^* (\hat{\tau}_1 - \hat{\tau}_6), \text{var}_d^* (\hat{\tau}_2 - \hat{\tau}_3), \text{var}_d^* (\hat{\tau}_2 - \hat{\tau}_4), \text{var}_d^* (\hat{\tau}_2 - \hat{\tau}_6), \text{var}_d^* (\hat{\tau}_2 - \hat{\tau}_7), \text{var}_d^* (\hat{\tau}_3 - \hat{\tau}_4), \text{var}_d^* (\hat{\tau}_3 - \hat{\tau}_5), \text{var}_d^* (\hat{\tau}_3 - \hat{\tau}_6), \text{var}_d^* (\hat{\tau}_3 - \hat{\tau}_7), \text{var}_d^* (\hat{\tau}_4 - \hat{\tau}_5), \text{var}_d^* (\hat{\tau}_4 - \hat{\tau}_6), \text{var}_d^* (\hat{\tau}_4 - \hat{\tau}_7), \text{var}_d^* (\hat{\tau}_5 - \hat{\tau}_6), \text{var}_d^* (\hat{\tau}_5 - \hat{\tau}_7), \text{var}_d^* (\hat{\tau}_6 - \hat{\tau}_7) \}$$

From Uddin (2008b) $d^*_1 \in D_{n=7}$ to denote the design having $n$ copies of the blocks (0,1,6,2,5,3,4); (1,2,0,3,6,4,5); (2,3,1,4,0,5,6); (3,4,2,5,1,6,0); (4,5,3,6,2,0,1); (5,6,4,0,3,1,2) and $(n+1)$ copies of the blocks (6,0,5,1,4,2,3).

5.3 Construction of Nearest Neighbour Balanced Block Designs for Seven Treatments

We have investigated MV optimality of NNBD when observations within a block are assumed to be correlated and generalized least square method of estimation has been used, when the error follows ARMA (1,1) models with $\nu = 7$, $b = 7$, $r = 7$, $k = 7$, $\lambda = 2$. Example,

Block: 1 (0,1,6,2,5,3,4)
Block: 2 (1,2,0,3,6,4,5)
Block: 3 (2,3,1,4,0,5,6)
Block: 4 (3,4,2,5,1,6,0)
Block: 5 (4,5,3,6,2,0,1)
Block: 6 (5,6,4,0,3,1,2)
Block: 7 (6,0,5,1,4,2,3)

6. MV- Optimality of Nearest Neighbour Balanced Block Design using First order Autoregressive Moving Average model

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$D_1$</th>
<th>$D_1^*$</th>
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</thead>
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<tr>
<td>0.1</td>
<td>1.22323</td>
<td>0.649136</td>
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<tr>
<td>0.2</td>
<td>1.80736</td>
<td>0.584916</td>
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<tr>
<td>0.3</td>
<td>2.55474</td>
<td>0.570698</td>
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<tr>
<td>0.4</td>
<td>3.56382</td>
<td>0.641742</td>
</tr>
<tr>
<td>0.5</td>
<td>4.99087</td>
<td>0.848973</td>
</tr>
<tr>
<td>0.6</td>
<td>20.0165</td>
<td>1.285215</td>
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<tr>
<td>0.7</td>
<td>185.996</td>
<td>2.179810</td>
</tr>
<tr>
<td>0.8</td>
<td>2132.221</td>
<td>4.247330</td>
</tr>
<tr>
<td>0.9</td>
<td>53826.81</td>
<td>309.6019</td>
</tr>
</tbody>
</table>

7. Conclusion

From table 6.1, we conclude that the MV-Optimality of Nearest Neighbour Balanced Block Design using First order autoregressive moving average model ARMA(1,1) shows the variance of the treatment differences for $D_1^*$ is less than $D_1$ for $\rho = 0.1, 0.2, 0.3, \ldots, 0.9$ under ARMA (1,1) model, therefore, we conclude that the design $D_1^*$ is MV- Optimal comparing with $D_1$. 

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8. References


