COMPARISON OF EXACT AND APPROXIMATE SOLUTION USING GALERKIN FINITE ELEMENT METHOD IN HIGHER DEGREE OF FREEDOM

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ABSTRACT

The higher degree of freedom while forming mathematical modeling and framing stiffness matrixes to define the problem using Galerkin Finite Element Method in structural engineering becoming complex. The author visualized and analyses the mathematical modeling of single degree of freedom and obtaining the linear variation with exact and approximate , however for two degree of freedom the variation of the solution become quadratic in nature , represented with the exact and approximate solution using Galerkin Finite element method . Increasing order of the degree of freedom and formation of mathematical modeling in analyzing the problem being represented in the form of cupid becoming new trend in case of three degree of freedom and comparison of exact and approximate analysis can be made with various conditions. Similarly, the higher order of degree of freedom in generalized Galerkin finite element method as such for fifth, sixth and seventh degree of freedom the formation of exact and approximate solution will be in fetid, sixid etc becoming new trend. It is required to develop the new trend in the computer analysis while designing the structural engineering domain element parameter using Galerkin based numerical analysis and formation of the stuffiness matrixes and solving the problem for the exact & Galerkin equation yields analysis of the problems.

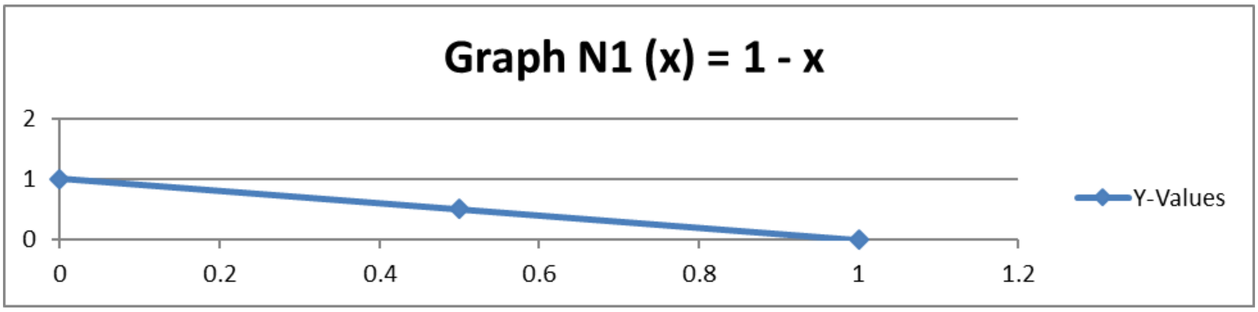
INDEX TERM: Mathematical modeling, Finite element method, Exact & Galerkin analysis, Two degree of freedom, Three degree of freedom, Non linear & linear behavior of mathematical model

I.PROBLEM INTRODUCTION

. The Finite Element Method in structural engineering becoming complex in analyzing the problem in generalized version it is required to develop the new trend in the computer analysis while designing the structural engineering domain element parameter. it becomes necessary to visualize the parametric analysis of the domain using Glarkins based numerical analysis and formation of the stuffiness matrixes and solving the problem for the exact & **Galerkin equation yields** analysis of the problems**.** The recent trend of **Gale kin**'s Matrix based Galerkin and exact analysis has been made for higher degree of freedom where as higher degree of freedom is complex and due to complexity the present technical dealt with the higher degree of freedom for analyzing exact and Galerkin problem is the basic concern. The exact and **Galerkin** solution simulate the non linear & linear behavior of the problem

II.INTRODUCTION AND PROBLEM IDENTIFICATION

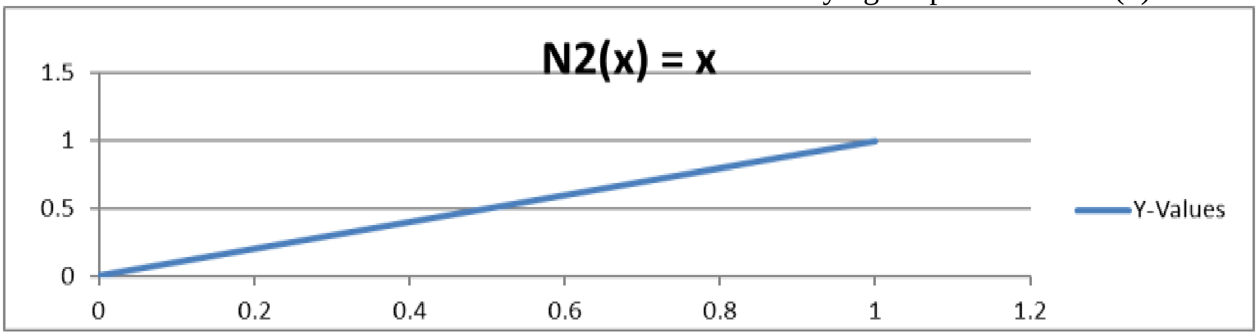
a) Problem identification and formulation: In formulation and solution of the Galerkin problem with one degree of freedom Representing n as number of degree of freedom = 1 , w h = c1 N1 and u h + qh = d1 N1 + q N2 , The only unknown is one and the shape function satisfying with N1(1) = 0, and N2(1), Let us take an example of one degree of freedom , N1 (x) = 1 - x and N2 (x) = x , If N1(x=1) =1-1 =0 and N2(x= 1) = 1 based on the above shape function , the assumed one degree of freedom satisfied .



Graph No1 Equation satisfying the assumed shape function , N(x) =0

N1(x)	0	.2	.4	.6	.8	1
1-x	1	.8	.6	.4	.2	0

Table No.1 Variation satisfying shape function N(x) =0



Graph No2 Equation satisfying the assumed shape function , N(x)= 1

N2(X)	0	.2	.6	1
X	0	.2	.6	1

Table No.2 Variation satisfying shape function N(x) =0

Result of the one degree of freedom:

i) Introduced the shape function , representing one degree of freedom
ii) Assumed mathematical modeling based on the shape function satisfied
iii) The variation of the function is becoming linear

b) The higher degree of freedom i.e second degree of freedom expressing as n = 2 , w h = c1 N1 + C2 N2 where shape function representing as N(1) = N(2) =0 , Uh = d1 N1 +d2 N2 + q N 3 , the unknown displacement d1 and d2 , where N3(1) =1
Let us define NA' S as follows

A)Mathematical modeling

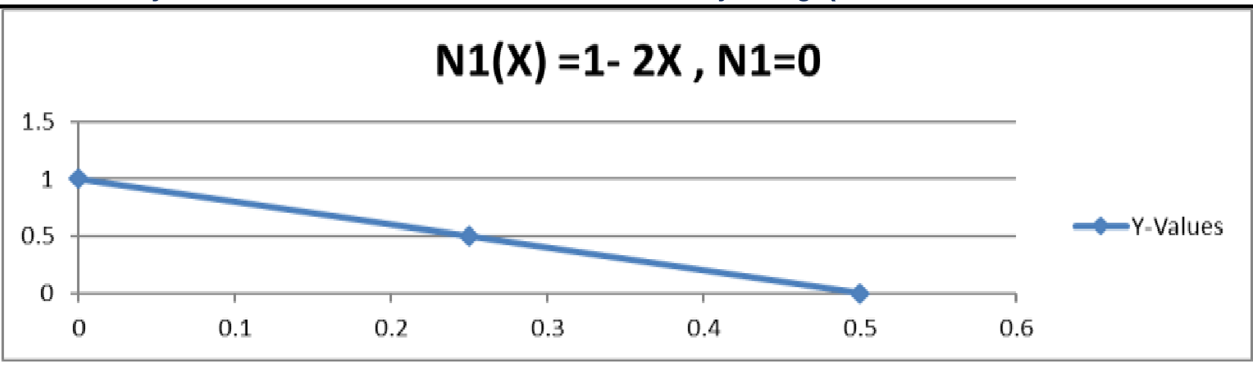
i)N1(x) = 1 – 2x $0 \leq x \geq .5$		
N1(0)	N1(x) = 1	
N1(.5)	N1(.5)= 0	

Table No 3 , Assumed equation for the adopted shape function

Mathematical modeling

ii)N1(x) = 0 $.5 \leq x \geq 1.0$
N1(.5) = 0
N1(1) = 0

Table No 4 , Assumed equation for the adopted shape function



Graph No 3 variation of graph with assumed equation to satisfy shape function

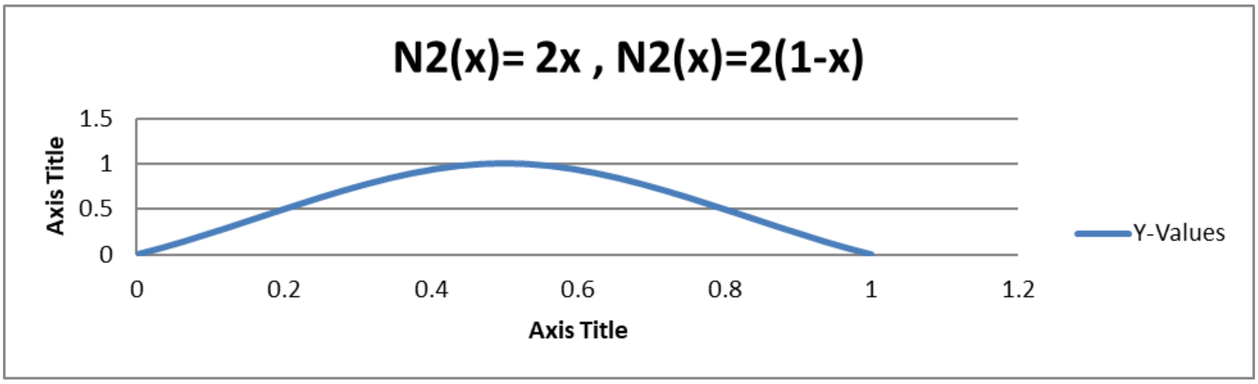
N1(x)	0	0.1	.2	.3	.4	.5
1-2x	1	.8	.6	.4	.2	0

Table No 5 graphical representation as per assumed equation

B) Mathematical modeling

N2(x) = 2x $0 \leq x \geq .5$,
N2(x) = 2(1 - x) $.5 \leq x \geq 1.0$

Table No 6 Assumed equation for shape function



Graph No 4 variation of graph with assumed equation to satisfy shape function

N2(x)	0	.2	.4	.5
2x	0	.4	.8	1.0

Table No 7 variation as per functional value of X with limitation

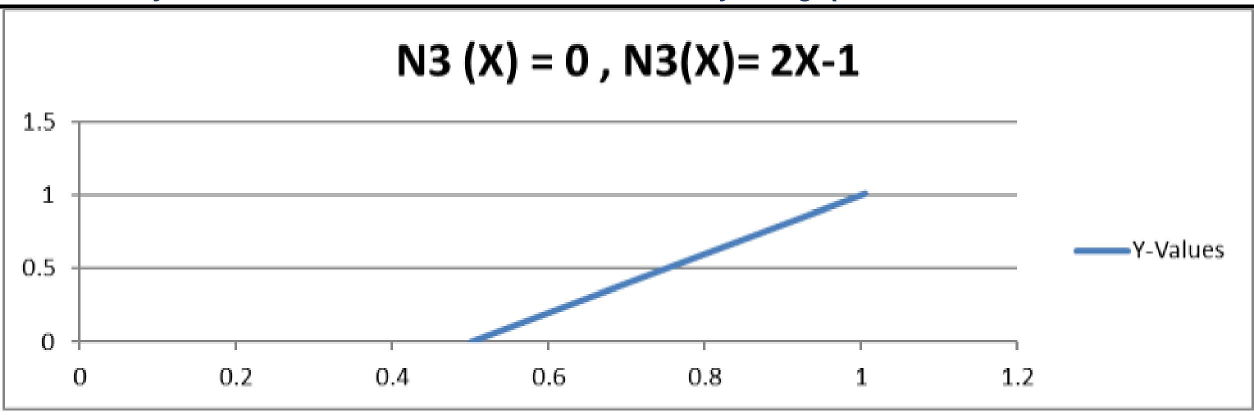
N2(x)	.5	.6	.8	1.0
2(1-x)	1.0	.4	.8	1.0

Table No 8 variation as per functional value of X with limitation

C) Mathematical modeling

N 3(x) =0	$0 \leq x \geq .5$,
N 3(x) =2x-1	$.5 \leq x \geq 1.0$

Table No 9 Assumed equation for satisfying



Graph No 5 variation of graph with assumed equation to satisfy shape function

N 3(x)	0	.2	.4	.5
0	0	0	0	0

Table No 10 Assumed equation for satisfying

N 3(x)	.5	.6	.8	1
2x-1	0	.2	.6	1

Table No 11 Assumed equation for satisfying

The second degree of freedom

Result and discussion

i) The variation of mathematically modelling representing in case of two degree of freedom become quadratic and the variation is being visualized in the second graph.

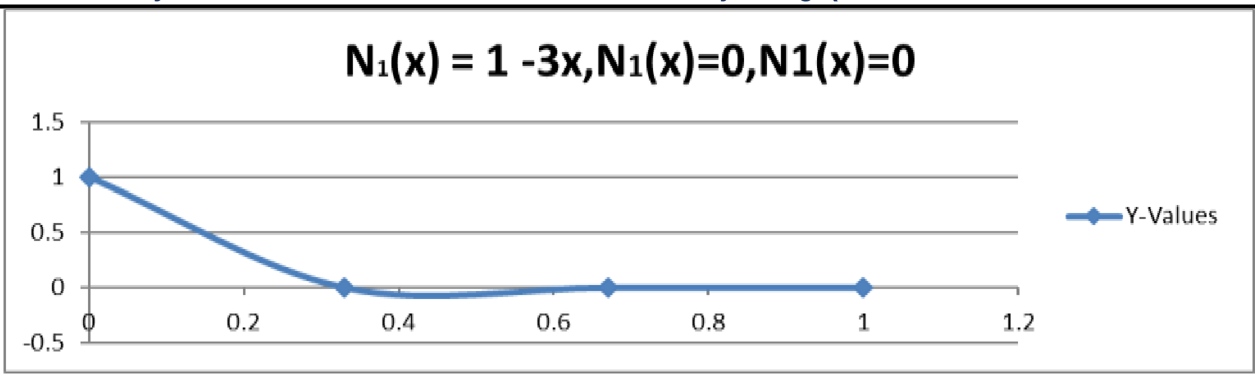
ii) The third graph shows the trend of the quadratic after the .5 ordinates .

C) Solving the third degree of freedom expressing as n = 3 , w h = c1 N1 + C2 N2 + C3N3 where shape function representing as N(1) = N(2) =N(3) =0 , Uh = d1 N1 +d2 N2 + d3N3 +q N 4 where N4(1) =1

Let us define NA' S as follows

N1(x) = 1 – 3x $0 \le x \ge 1/3$,
N1(x) = 0 $.33 \le x \ge .67$,
N1(x) = 0 $.67 \le x \ge 1.0$
N2(x) = 3x $0 \le x \ge 1/3$,
N2(x) = 3(1 -3x) $1/3 \le x \ge 2/3$,
N2(x) = 3(1 - x) $2/3 \le x \ge 1.0$,
N 3(x) =0 $0 \le x \ge 1/3$,
N 3(x) =3x-2 $1/3 \le x \ge 2/3$
N 3(x) =0 $2/3\le x \ge 1.0$,
N4(x) =0 $0 \le x \ge 1/3$,
N4(x) =0 , $1/3 \le x \ge 2/3$,
N 4(x) =3x-2 ,$2/3\le x \ge 1.0$

Table Number 12 Assumed Equation to satisfy the existing shape function



Graph No 6 variation of graph with assumed equation to satisfy shape function

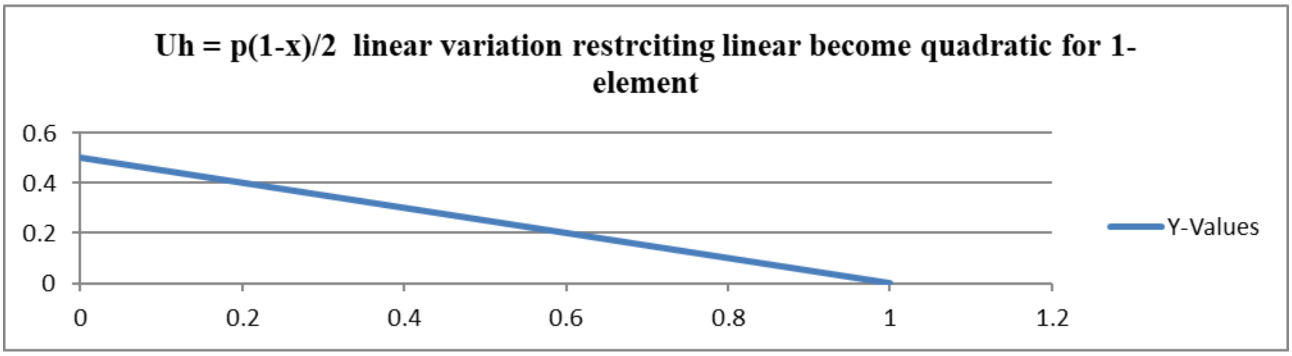
D) Solving the fourth degree of freedom expressing as n = 4 , w h = c1 N1 + C2 N2 + C3N3 + C4N4 where shape function representing as N(1) = N(2) =N(3) = N(4)=0 , Uh = d1 N1 +d2 N2 + d3N3 + d4N4+q N 5 where N5(1) =1 . The equation satisfying the shape function can be established and nature of graph can be predicted.
E) Solving the fourth degree of freedom expressing as n = 5, w h = c1 N1 + C2 N2 + C3N3 + C4N4 + C5N5 where shape function representing as N(1) = N(2) =N(3) = N(4)=N(5)=0 , Uh = d1 N1 +d2 N2 + d3N3 + d4N4+N5d5 + q N 5 where N5(1) =1 . The equation satisfying the shape function can be established and nature of graph can be predicted.Table Number 12 Assumed Equation to satisfy the existing shape function

III.FORMULATION OF GALERKIN EQUATION YIELDS

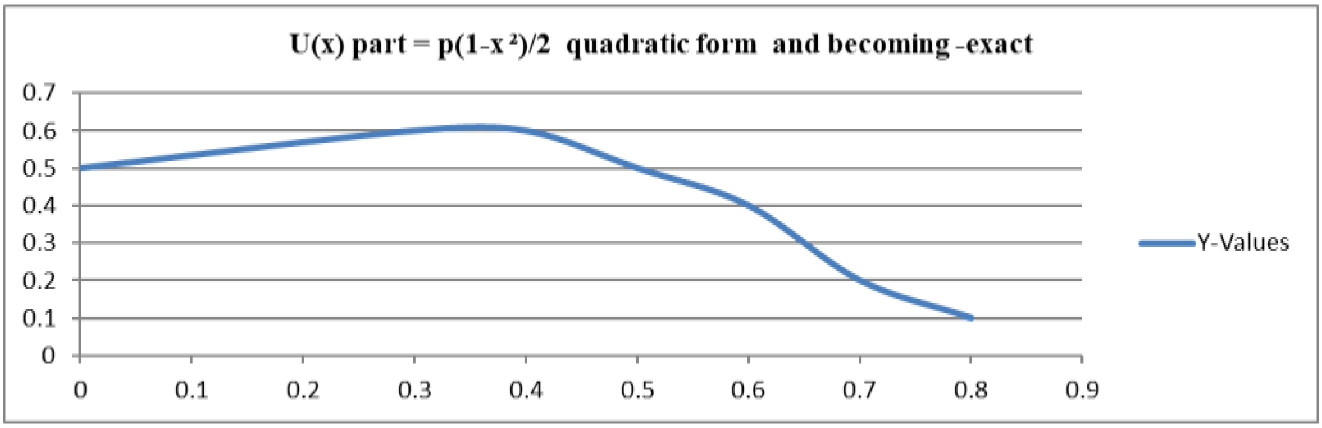
a(∑cA NA ∑dBNB) = ∑cA NAℓ+(∑cA NA)h – a(∑cA NAqNn+1) ,By using bi linearity of a(…) and (…) the Galerkin equation yields becomes 0 = ∑CA GA , GA =∑a (NA NB)dB – (NA, ℓ) – NA(0)h + a(NA NB)q
Stiffness matrix : $(\{K\})[X] = \{F\}$

$$\mathbf{K = 2 *} \begin{matrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{2} \end{matrix}$$

KAB =a (NA ,NB) = ∫NA NB dx = ∫ NA.x N B.x dx
K11 = 2 , k12 = K21 = -2 , K 22 = 4
F1 = N1 **ℓ** + N1(0) h – a(N1 ,N2) **q** = ∫(1-x) **ℓ x dx** + h -∫ N1.x ,N2.x dx **q**
N1.x = 1- x = -1 , N2.x = x = +1 , F1 =∫(1-x) **ℓ x dx** + h -∫ dx **q**
d= K^{-1} F , u h = (∫(1-x) ℓ x dx + h -∫ dx q)*(1 – x))/d1 + q x
where y plays an important role while getting a nature of approximation and shows an variables . Let compare with the exact solution .
Case i Let us try for ℓ = 0 , u (x) = q + (1 – x) h it has been visualized with exact solution if the solution Exact solution u (x) = q + (1 – x) h + ∫ ℓ (z) dz dy where u xx + ℓ = 0 i.e. u xx (x) + ℓ (x) = 0 for all x Є Ω if source term ℓ becoming zero the solution becoming exact i.e. the approximate solution become the exact solution . The approximate pertain to particular solution i.e. the part of the solution become ℓ is non zero .
Case ii If source term ℓ becoming non zero ,assume ℓ(x) = ρ a constant then particular solution become
U part(x) = (1- x^2) /2 & U h part (x) = (1- x) /2



Graph 7 Linear variation restricting linear become quadratic



Graph 8 Quadratic form and becoming –exact

IV. RESULT AND DISCUSSION

i)In the analysis of Galerkin problem it is first to declare the degree of the freedom. The present case of analysis is subjected to define the number of the unknown and the number of the unknown is becoming 1° is d1.

ii)The degree of freedom for the first set of domain is becoming wh= C1N1 and uh + qh = d1N1 + q N2 . The above set of the formulation one degree of freedom i.e. value become d1 .

iii)The only unknown is one and the shape function satisfying with N1(1) = 0, and N2(1), Let us take an example of one degree of freedom , N1 (x) = 1 - x and N2 (x) = x , If N1(x=1) =1-1 =0 and N2(x= 1) = 1 based on the above shape function , the assumed one degree of freedom satisfied .

iv)In the assumed equation referring to the adopted shape function and tabular chart no 1 representing the variation of the graph as represented become linear expressed in graph no 1 and the shape function have been represented by shape function , N(x) =0

v)In the assumed equation referring to the adopted shape function and tabular chart no 2 representing the variation of the graph as represented become linear expressed in graph no 2 and the shape function have been represented by shape function , N(x) =x

vi) The higher degree of freedom i.e. 2 ° of freedom whose equation expressing as w h = C1 N1 + C2 N2 where shape function representing as N(1) = N(2) =0 , Uh = d1 N1 +d2 N2 + q N3 , the unknown displacement d1 and d2 representing 2 ° of freedom .

vii) The higher degree of freedom i.e second degree of freedom expressing as n = 2 , w h = c1 N1 + C2 N2 where shape function representing as N(1) = N(2) =0 , Uh = d1 N1 +d2 N2 + q N 3 , the unknown displacement d1 and d2 , where N3(1) =1

viii)The expressed shape function of the equation must satisfy the existing equation ,N1(x) = 1 – 2x with limitation $0 \leq x \geq .5$, if the value of the x being placed as 0 , result of assumed equation become 1 and when it being represented with the highest limitation as .5 , the value become 0. That is what the basis of adopted value of the shape function .

ix) The expressed shape function of the equation must satisfy the existing equation ,N1(x) =0 with limitation $0.5 \leq x \geq 1.0$, if the value of the x being placed as .5 , result of assumed equation become 0 and when it being represented with the highest limitation as 1.0 , the value become 0. That is what the basis of adopted value of the shape function .

x) The expressed shape function of the equation must satisfy the existing equation ,N2(x) = 2x with limitation $0 \leq x \geq .5$, if the value of the x being placed as 0 , result of assumed equation become 0 and when it being represented with the highest limitation as .5 , the value become 1. That is what the basis of adopted value of the shape function. The variation of shape function in the higher degree of freedom changing the nature of the graph i.e. the graph have been visualize as quadratic instead of linear as shown in graphical form i.e. quadratic one in graph number 4.

xi) The expressed shape function of the equation must satisfy the existing equation, N2(x) = 2(1-x), with limitation $.5 \leq x \geq 1.0$, if the value of the x being placed as .5 , result of assumed equation become 1 and when it being represented with the highest limitation as 1.0 , the value become 0. That is what the basis of adopted value of the shape function .Same have been shown in graphical form i.e. quadratic one in graph number 4.

xii) The domain is define with mathematical modeling considering the degree of freedom as three as such the basic equation become w h = C1 N1 + C2 N2 + C3N3 where shape function representing as N(1) = N(2) =N(3) =0 , Uh = d1 N1 +d2 N2 + d3N3 +q N 4 where N4(1) =1

xiii) The domain is define with mathematical modeling considering the degree of freedom as three as such the basic equation become d1 , d2 , d3. Now the assumed equation have to satisfy the shape function.

xv) The expressed shape function of the equation must satisfy the existing equation, N1(x) = 1-3x, with limitation $0 \leq x \geq 1/3$, if the value of the x being placed as 0 result of assumed equation become 1 and when it being represented with the highest limitation as 1/3 , the value become 0. That is what the basis of adopted value of the shape function .Same have been shown in graphical form i.e. cubical one in graph number 5.

xvi) The expressed shape function of the equation must satisfy the existing equation, N1(x) = 0, with limitation $.33 \leq x \geq .67$, if the value of the x being placed as 0 result of assumed equation become 0 and when it being represented with the highest limitation as .67 , the value become 0. That is what the basis of adopted value of the shape function .Same have been shown in graphical form i.e. quadratic cubical one in graph number 5.

xvii) The expressed shape function of the equation must satisfy the existing equation, N1(x) = 0, with limitation $0 \leq x \geq 1/3$, if the value of the x being placed as .67 result of assumed equation become 0 and when it being represented with the highest limitation as 1 , the value become 0. That is what the basis of adopted value of the shape function .Same have been shown in graphical form i.e. quadratic one in graph number 4.

xviii) The expressed shape function of the equation must satisfy the existing equation, N2(x) = 3x, with limitation $0 \leq x \geq 1/3$, if the value of the x being placed as 0 result of assumed equation become 0 and when it being represented with the highest limitation as 1/3 , the value become 1. That is what the basis of adopted value of the shape function .Same have been shown in graphical form i.e. quadratic one in graph number 4.

xix) The expressed shape function of the equation must satisfy the existing equation, N2(x) = 3(1-3x), with limitation $1/3 \leq x \geq 2/3$, if the value of the x being placed as 1/3 result of assumed equation become 0 and when it being represented with the highest limitation as 2/3 , the value become 0. That is what the basis of adopted value of the shape function .Same have been shown in graphical form i.e. quadratic one in graph number 4.

xx) The expressed shape function of the equation must satisfy the existing equation, N2(x) = 3(1-3x), with limitation $2/3 \leq x \geq 1$, if the value of the x being placed as 2/3 result of assumed equation become 0 and when it being represented with the highest limitation as 2/3 , the value become less than 0. That is what the basis of adopted value of the shape function .Same have been shown in graphical form i.e. quadratic one in graph number 4.

xxi) The expressed shape function of the equation must satisfy the existing equation, N3(x) = 3x-2, with limitation $0 \leq x \geq 1/3$, if the value of the x being placed as 0 result of assumed equation become 0 and when it being represented with the highest limitation as 1/3 , the value become less than 1. That is what the basis of adopted value of the shape function .Same have been shown in graphical form i.e. quadratic one in graph number 4.

xxii) The expressed shape function of the equation must satisfy the existing equation, N3(x) = 3x-2, with limitation $1/3 \leq x \geq 23$, if the value of the x being placed as1/3 result of assumed equation become 1 and when it being represented with the highest limitation as 2/3 , the value become less than 0. That is what the basis of adopted value of the shape function .Same have been shown in graphical form i.e. cubical nature of graph number 4.

xxiii) The expressed shape function of the equation must satisfy the existing equation, N3(x) = 3x-2, with limitation $2/3 \leq x \geq 1$, if the value of the x being placed as 2/3 result of assumed equation become less than 0 and when it being represented with the highest limitation as 1 , the value become 1. That is what the basis of adopted value of the shape function .Same have been shown in graphical form i.e. cubical nature of graph number 4.

xxiv) The expressed shape function of the equation must satisfy the existing equation, N4(x) = 0, with limitation $0 \leq x \geq 1/3$, if the value of the x being placed as 0 result of assumed equation become 0 and when it being represented with the highest limitation as 1/3 , the value become 0. That is what the basis of adopted value of the shape function .

xxv) The expressed shape function of the equation must satisfy the existing equation, N4(x) = 0, with limitation $2/3 \leq x \geq 1$, if the value of the x being placed as 2/3 result of assumed equation become less than

0 and when it being represented with the highest limitation as 1 , the value become 1. That is what the basis of adopted value of the shape function .

xxvi) The expressed shape function of the equation must satisfy the existing equation, N4(x) = 0, with limitation 1/3 ≤ x ≥ 2/3, if the value of the x being placed as 1/3 result of assumed equation become less than 0 and when it being represented with the highest limitation as 2/3 , the value become 0. That is what the basis of adopted value of the shape function .

xxvii) The expressed shape function of the equation must satisfy the existing equation, N4(x) = 0, with limitation 1/3 ≤ x ≥ 2/3, if the value of the x being placed as 1/3 result of assumed equation become less than 0 and when it being represented with the highest limitation as 2/3 , the value become 0. That is what the basis of adopted value of the shape function .

xxviii) In case fourth degree of freedom expressing as n = 4 , the equation become w h = c1 N1 + C2 N2 + C3N3 + C4N4 where shape function representing as N(1) = N(2) =N(3) = N(4)=0 , Uh = d1 N1 +d2 N2 + d3N3 + d4N4+q N 5 where N5(1) =1 , based on the shape function and the nature of graph can be visualized and may obtained in nature of fourtic .This is basis of further research and define the analysis with the exact analysis .

xxix)) In case fifth degree of freedom expressing as n = 5 , w h = c1 N1 + C2 N2 + C3N3 + C4N4 + C5N5 where shape function representing as N(1) = N(2) =N(3) = N(4)=N(5)=0 , Uh = d1 N1 +d2 N2 + d3N3 + d4N4+N5d5 + q N 5 where N5(1) =1 , based on the shape function and the nature of graph can be visualized and may obtained in nature of fistic . The equation satisfying the shape function can be established and nature of graph can be predicted.

V. CONCLUSION

i)In the analysis of Galerkin problem it is first to declare the degree of the freedom. The present case of analysis is subjected to define the number of the unknown and the number of the unknown is becoming 1° is d1.The degree of freedom for the first set of domain is becoming wh= C1N1 and uh + qh = d1N1 + q N2 . The above set of the formulation one degree of freedom i.e. value become d1 .

ii)Introduced the shape function, representing one degree of freedom and two degree while representing one degree of freedom the linear variation and two degree of freedom quadratic variation .

iii) The mathematical modeling based on the shape function satisfying whole keeping numerical values of the aforesaid assumed modeling is the prime requirement of the finite element method with exact analysis

iv)The variation of the function is becoming linear or quadratic or cubic or for tic or fistic or six tic nature of variation can be visualized by plotting the assumed modeling with the shape function .

v) In case fourth degree of freedom , on the basis the shape function and the nature of graph can be visualized and may obtained in nature of four tic .This is basis of further research and define the analysis with the exact analysis .The basic requirement of comparing the exact and approximate analysis of behavior of model.

vi) In case fourth degree of freedom , on the basis the shape function and the nature of graph can be visualized and may obtained in nature of fistic .This is basis of further research and define the analysis with the exact analysis .The basic requirement of comparing the exact and approximate analysis of behavior of model.

vii) For all x Є Ω if source term ℓ becoming zero the solution becoming exact i.e. the approximate solution become the exact solution . The approximate pertain to particular solution i.e. the part of the solution become ℓ is non zero .

viii). If source term ℓ becoming non zero ,assume ℓ(x) = ρ a constant then particular solution become U part(x) = (1- x /2 & U h part (x) = (1- x) /2 & with the graph data 1 , variation become linear restricting linear become quadratic for one element

ix). If source term ℓ becoming non zero ,assume ℓ(x) = ρ a constant then particular solution become U part(x) = (1- x^2 /2 & U h part (x) = (1- x^2) /2 & with the graph data 2 , variation become become quadratic –become exact one

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