A Method for Reducing Secondary Field Effects in Asymmetric MRI Gradient Coil Design

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Abstract: This research introduces an original method for the design of MRI gradient coils that reduces secondary field effects created by eddy current coupling. The method is able to deal with asymmetric coils and provides a new way to ensure a reduction in the magnitude of the eddy current induced fields. New constraints are introduced at the surface of passive objects to bind the normal field component below a given value. This value is determined by first treating the passive surface as an active surface and then calculating the ideal stream function on that surface to produce the desired secondary field. Two coils were designed, one to image the knee and the other to image the head and neck. The power loss in the passive structure also decreased to below 1% of the original value using the new method. The method shows the ability to constrain the field to values below the minimum seen under the traditional approaches.

This will allow the design of asymmetric systems with highly linear, reduced magnitude of secondary fields and may lead to better image quality.

Keywords: MRI, Secondary Fields, Gradient Coils, Power Loss

1. INTRODUCTION

Magnetic resonance imaging (MRI) makes use of gradient coils which superimpose a spatially varying magnetic field, onto the primary magnetic field in order to provide localization for the imaging of soft tissue samples [1]. Typically an MRI machine will include one gradient coil for each of the Cartesian axes (x, y, and z). These coils provide a uniform magnetic gradient within a small volume referred to as the region of uniformity (ROU). Fig. 1 and 2 Shows diagrammatic representation of the major components of an MRI system as well as an example of current pathways for an actively shielded asymmetric gradient coil.
At the core of gradient coil design is a trade-off between the desired magnetic field and competing parameters including resistance and inductance. Various methods have been created in order to assist in the design of these coils. These methods can be roughly broken into two categories, analytic models and mesh based models. Analytic solutions are usually restricted to simple, parametrisable, surfaces and include techniques such as Fourier methods and target field methods [2]. Mesh based methods discretize surfaces into small, typically triangular, elements and form a basis function expressed at the nodes of each element.

Figure 1: Diagram of MRI layout showing major components within the MRI scanner
Figure 2: Conductive pathways on a cylindrical surface used to induce a magnetic gradient useful for MRI (i.e. \( G_y = \frac{dB_z}{dy} \)).

Optimization parameters such as the magnetic field, resistance and inductance are then calculated on this basis. When paired with the correct boundary conditions, this formulation can be suitably used for optimization using techniques such as quadratic programming [2, 3, 4]. Induced in conductive material in the presence of a time varying magnetic field, eddy currents cause a number of issues including power loss, increased heating and a secondary magnetic field [5]. This secondary field can combine with the field generated by the gradient coil and disrupt the gradient profile [6]. These coils are used in the imaging of the neck and other peripheral regions [7, 8]. In some coils part of the ROU may lie close to or beyond the edge of the surface.

This research aims to investigate the eddy current behavior in asymmetric gradient coils through numerical simulation. Once this model has been established a new method of design will be presented to help deal with this behavior. This goal is to match the eddy current and primary magnetic field profile and to minimize power losses within the passive structure. The work builds on methods for design and simulation which have been shown to correctly model the behavior of eddy currents in thin conductors [9, 10].
2. METHODS

2.1 Modeling

Let us consider a homogeneous unbounded 3D domain with filamentary coils with known currents and a conductive region $V$. Assuming magnetic quasi-static approximation, the current density $J$ in $V$ is divergence free and can be described with the electric vector potential $T$. The study is restricted to the case when $V$ can be approximated by 2D curved surfaces $S$, i.e. the current density lies on these surfaces.

In this case the vector potential can be represented as the scalar stream function, defined as $T = n(r)\psi(r)$ [11], where $n$ is the unit normal to the current density plane.

The stream function is expanded by nodal shape functions $\lambda_k$ so that the current is interpolated by piece-wise constant vector basis functions $f_k(r)$:

$$J(r) = \sum_{k=1}^{N} \psi_k \nabla \times \left( \lambda_k(r)n(r) \right) = \sum_{k=1}^{N} \psi_k f_k(r)$$ (1)

Where $\psi_k$ is the value of the stream function at the $N$ mesh nodes.

![Figure 3: Vector basis function (grey lines) $f_i(r)$ associated to the node $i$ of the mesh.](image)

In case of triangular discretization, in the generic $\alpha$th triangle having $k$ as vertex, it can be proved that:

$$f_k^{(\alpha)}(r) = \nabla \times \left( \lambda_k^{(\alpha)}(r)n^{(\alpha)}(r) \right) = \frac{1}{2S^{(\alpha)}} e_k^{(\alpha)}$$ (2)

$e_k$ is the vector corresponding to the edge opposite to node $k$ (see Figure 3), and $S^{(\alpha)}$ is the triangle area. Figure 3 shows a graphical representation of the vector shape function $f_k$.

Using the Green’s function the electric field in the frequency domain can be expressed as:

$$\frac{J(r)}{\delta(r)\sigma(r)} + j\omega \frac{\mu_0}{4\pi} \int \frac{J(r')}{|r-r'|} ds + \nabla \varphi = -j\omega A_s(r)$$ (3)

Being $r'$ the vector pointing to the source point, $r$ the vector pointing to the field point and $A_s(r)$ the magnetic vector potential due to the known sources in $V$. $\delta(r)$ and $\sigma(r)$ are the thickness and conductivity of the surface $S$.

The equality is imposed in average over the entire surface domain $S$, using $f_j(r)$ as test function, resulting in a Galerkin scheme:
\[
\sum_{k=1}^{N} (R_{jk} + j\omega L_{jk})\psi_k = -j\omega a_{s,j} \tag{4}
\]

Where the resistive, inductive and source terms are, respectively:

\[
R_{jk} = \int \frac{1}{\delta(r)\sigma(r)} f_j(r) \cdot f_k(r) \, ds
\]

\[
L_{jk} = \frac{\mu_0}{4\pi} \int f_j(r) \cdot \int \frac{1}{|r-r'|} f_k(r') \, ds \, ds
\tag{5}
\]

\[
a_{s,j} = \int f_j \cdot A_s(r) \, ds
\]

If the number of test functions \( f_j(r) \) is chosen equal to the number of nodal unknowns, a \( N \times N \) system of linear equation is obtained. The magnetic flux density is then calculated directly from the stream function value.

### 2.2 Traditional Method

The process of generating an ideal stream function solution is a minimization problem outlined as:

Minimize \( \lambda_1 \psi' R \psi + \lambda_2 \psi' L \psi \)

Subject to \( B_t - \alpha \psi < B_z \psi < B_t + \alpha \)

\[
B_c - \beta < B_s \psi < B_c + \beta \tag{6}
\]

\[
\gamma_{x,y,z} < T_{x,y,z} \psi < \gamma_{x,y,z}
\]

Where \( \lambda_1 \) and \( \lambda_2 \) are optimisation weighting factors, \( \alpha_1 \) is an error weighting factor, \( B_z(r) \) is the \( z \) component of the target field within the ROU, \( M_z \) is the magnetisation matrix for each target point within the ROU, \( B_s \) is the field limit at shielding target points, and \( M_s \) is the magnetization matrix calculated at these points. As the matrices \( L \) and \( R \) contain values of differing units some normalization must occur.

### 2.3 Coil Performance Metrics

To test these methods two coils were designed and numerically evaluated under both the traditional and new methods. The first coil was chosen to provide imaging of the knee in an asymmetric system. In this coil the ROU was chosen to be an ellipsoid with a radius of 75 mm along the axial direction and a radius of 120 mm along the \( z \) direction. The entire ROU was shifted 90 mm along the \( z \) direction. For the purposes of evaluating the spherical harmonics present within the field an additional spherical ROU of radius 75 mm was used. The size of the surfaces, two active and one passive, are defined in Table 1. All surfaces had a thickness of 2.5 mm. Figure 4 shows the layout of the surfaces and the ROU.

Table 1: Knee Coil Dimensions and Properties

<table>
<thead>
<tr>
<th>Surface</th>
<th>Radius(mm)</th>
<th>Length(mm)</th>
<th>Resistivity((\Omega m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>110</td>
<td>624</td>
<td>1.68(e^{-8})</td>
</tr>
<tr>
<td>Secondary</td>
<td>126</td>
<td>624</td>
<td>1.68(e^{-8})</td>
</tr>
<tr>
<td>Passive</td>
<td>152.5</td>
<td>660</td>
<td>2.82(e^{-8})</td>
</tr>
</tbody>
</table>
Figure 4: Knee Coil System Layout

The second coil was designed to image the head and neck region. This coil featured an ROU with a length of 160 mm along the $z$ direction and a radius of 120 mm. The entire ROU was shifted 70 mm from the centre along the axial direction. For the evaluation of spherical harmonics a spherical ROU with a radius of 80 mm was used.

Table 2: Head and Neck Coil Dimensions and Properties

<table>
<thead>
<tr>
<th>Surface</th>
<th>Radius (mm)</th>
<th>Length (mm)</th>
<th>Resistivity ($\Omega m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>150</td>
<td>480</td>
<td>$1.68 \times 10^{-8}$</td>
</tr>
<tr>
<td>Secondary</td>
<td>172</td>
<td>480</td>
<td>$1.68 \times 10^{-8}$</td>
</tr>
<tr>
<td>Passive</td>
<td>208</td>
<td>508</td>
<td>$2.82 \times 10^{-8}$</td>
</tr>
</tbody>
</table>
Figure 5: Head and Neck Coil System Layout

3. RESULTS AND DISCUSSION

All simulations were undertaken using Matlab R2013b on a machine with an Intel Xeon 3.5 GHz processor and 64 GB of RAM. Each of the three cylinders was discretized using 3080 triangles and 1610 nodes. The ROU was constrained using 642 points on the surface of the volume. Shielding was constrained using the nodes on the outer passive surface giving 1610 constrained points. Each coil was designed and modeled within 300 seconds.

Power Loss

Table 3 shows the calculated power loss in the passive structure using both methods. Here the power can be seen to be significantly reduced in both cases under the new method. This reduction in power loss can be critical to the performance of MRI systems.

Table 3: Power loss in the passive structure

<table>
<thead>
<tr>
<th>Coil</th>
<th>Method</th>
<th>Power loss (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knee</td>
<td>Traditional Method</td>
<td>46.20</td>
</tr>
<tr>
<td>Knee</td>
<td>SNFDP Method</td>
<td>0.088</td>
</tr>
<tr>
<td>Head</td>
<td>Traditional Method</td>
<td>70.75</td>
</tr>
<tr>
<td>Knee</td>
<td>SNFDP Method</td>
<td>0.082</td>
</tr>
</tbody>
</table>

Figure 6: Power loss in the passive structure under both methods (Knee coil)

Figure 6 shows the calculated power loss in the passive structure of the knee coil design. The power is, in all cases, reduced in the new method. The traditional method appears to reach a threshold, after which the power begins to increase again. This increase in power at low constraints was not observed in the new method over the tested range.
Figure 7: Power loss in the passive structure for a symmetric and asymmetric coil (Traditional Method)

Figure 7 shows the power loss under the traditional model for a symmetric and asymmetric knee coil. The symmetric coil has the same constraints as the asymmetric coil but with no offset to the ROU. This comparison highlights the difficulty of designing asymmetric coils. The rise in power loss at low constraints is not seen in the symmetric design.

Figure 8: Shielding ratio calculated under both methods (Knee coil)

Figure 8 shows the calculated shielding ratio for both methods. Again, the new method exhibits a decrease at all points. This decrease in shielding ratios indicates the lessened effects of eddy currents within the ROU, which helps to ensure images are free from distortion.

4. CONCLUSION
This work shows a new method to reduce the secondary field effects when designing asymmetric gradient coils. The method shows a reduction in secondary field strength as constraints decrease which is not seen in the traditional method. The coils produced using this method show a more feasible resulting design when constraints are limited to low values. This work allows asymmetric gradient coils to be designed with higher performance metrics. This is of interest to systems designed for the imaging of regions such as the knee or head and neck.

5. References


