RELIABLE SAMPLED DATA CONTROL FOR NEURAL NETWORKS WITH MIXED TIME DELAYS

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Abstract: This paper concerns the problem of reliable sampled data control for neural networks with time varying delays and actuator faults. By constructing a suitable Lyapunov-Krasovskii functional, sufficient conditions are derived to prove that the addressed neural networks is stable. The derived conditions are obtained in the form of LMIs. The controller gain matrix is obtained by solving the LMIs using the well known numerical MATLAB software. At last, numerical example is presented to show the effectiveness of the theoretical results.

Index Terms - Reliable, sampled-data control, actuator faults.

I. INTRODUCTION

During the past few years, neural networks have received much more attention due to a wide applications, such as signal processing, target tracking, image processing, associative memory, pattern recognition, static im age processing, optimization problems, power systems, finance, parallel computing, mechanics of structures, materials, smart antenna arrays and other scientific areas [1]-[3]. Some of diverse delayed neural networks such as Hopfield NNs, Cohen-Grossberg neural networks, cellular neural networks and bidirectional associative memory neural networks have been extensively investigated by many researchers. Simultaneously, the study on time delay neural networks has received great attention since time-delay is an inherent feature of many physical processes, such as chemical processes, nuclear reactors biological systems, and it may lead to instability or significantly deteriorated performances for the closed-loop systems. Normally, time delay is experienced in electronic executions of neural networks, in sight of the finite switching speed of the amplifiers and communication. Neural networks were discussed by many researchers in both the case of continuous and discrete [4]-[6]. Moreover, stability analysis of neural networks with time delay [7], time varying delay [8], interval time varying delay [27],[10], mixed delay [11] are investigated by many researchers due to its necessity in real life.

The most important properties of control systems is stability, by virtue of there is no practical applications for unstable systems. Fundamentally, that every control system is definitely stable and then the other properties can be studied. The stability analysis of neural networks can be expressed in linear matrix inequality which can be in [12],[13]. The stability of neural networks with various controls like $H_{\infty}$ control, sampled data control, reliable control, non-fragile control, impulsive control, event-triggered control can be examined by many researchers using the various integral inequality technique like Jensen’s inequality, Wirtinger based integral inequality etc., [14]-[19]. By virtue of the growing technology of automated control systems, some faults are entered that can be from actuators and sensors. A reliable control system acquires the capability to hold the system failures naturally and maintains that the closed loop system achieves stability. Stability analysis of reliable control for uncertain system with time varying delays is examined in [20]-[21].

Due to the rapid growth of technology, a controllers became widely, overall control systems becomes a sampled data system in which the control signals are in constant at the time of sampling period and then it will change at the sampling time which leads to the control signals are in stepwise, which means discontinuities exists and it assembles the dynamical systems in complicated position. Moreover, stability and stabilization of system with sampled-data control is investigated in [22]-[24].

Motivated by the above deliberations, utilization of both reliable and sampled data control for the stability of mixed delay neural networks is not yet fully studied. So this encourage me to focuses on the stability of mixed delay neural networks using reliable control with actuators failures and sampled data control.

This paper is organized as follows: The problem description and some of the essential Assumption, Definition and Lemmas are mentioned in section 2. In section 3, the main part of this paper i.e., the main results are given through Theorem and corollaries. A numerical example is given to show the sureness of the theoretical results in section 4. Finally, a conclusion part is given in section 5.

II. PROBLEM DESCRIPTION AND PRELIMINARIES

Consider a mixed delay neural networks in the following form

$$\dot{x}(t) = -Ax(t) + Bf(x(t)) + Cf\left(x(t-\tau(t))\right) + D \int_{t-\tau(t)}^{t} f(x(s)) \, ds + u^p(t)$$  

(1)

where $x(t) \in \mathbb{R}^m$ is state vector of network at time $t$, $f(x(t))$ is the neuron activation function at time $t$, $u^p(t)$ is the control input with actuator failures, $A = \text{diag}\{a_1, a_2, ..., a_n\} > 0$, $B = (b_{ij})_{n \times n}$, $C = (c_{ij})_{n \times n}$, $D = (d_{ij})_{n \times n}$ is known interconnection weight matrices between neurons, $\tau(t)$ is continuous time varying function and satisfies
where $\tau$ and $\rho$ are constants.

We consider the reliable control as

$$0 \leq \tau(t) \leq \tau^r(t) \leq \rho$$  \hspace{1cm} (2)

$$u^r(t) = Go(t)$$  \hspace{1cm} (3)

where $G$ is the actuator fault with $G = \text{diag}\{g_1, g_2, ..., g_m\}$, $0 \leq g_m \leq 1$, $m = 1, 2, ..., n$.

Now consider the sampled-data control

$$u(t) = k_x(t), \hspace{1cm} k = 0, 1, 2, ..., n \hspace{1cm} (4)$$

where $K \in R^{n \times n}$ is the sampled-data feedback controller gain matrix to be designed. $t_i$ denotes the sample time point and satisfies $0 = t_0 < t_1, ..., t_k < t_{k+1} < ...$ and $\lim_{k \rightarrow \infty} t_k$. Moreover, there exists a positive constant $h$ such that $t_{k+1} - t_k \leq h$, $\forall k \in N$. Let $h(t) = t - t_k$, for $t \in [t_k, t_{k+1})$, then $t_k = t - h(t)$ with $0 < h(t) \leq h$.

Under the control law (3) and (4), the neural networks (1) can be written as follows:

$$\dot{x}(t) = -Ax(t) + Bf(x(t)) + C \int_{t-	au(t)}^{t} f(x(s)) ds + Gk_x(t - h(t)). \hspace{1cm} (5)$$

**Assumption 2.1.**  \hspace{0.5cm} [25] For any $j \in 1, 2, ..., n$, $f_j = 0$ and there exist constants $F^+_j$ and $F^-_j$ such that

$$\frac{f_j(x(t)) - f_j(x(t))}{|x(t) - x(t)|} \leq F^+_j \hspace{1cm} \forall \alpha \neq \alpha.$$

**Definition 2.2.**  \hspace{0.5cm} [26] The system is said to be asymptotically stable if it is stable, and for any $t_0 \in R^n$ and any $\epsilon > 0$, there exists a $\delta_0 = \delta_0(t_0, \epsilon) > 0$ such that $\|x(t_0)\| < \delta_0$ implies $\lim_{t \rightarrow \infty} x(t) = 0$.

**Lemma 2.3.** (Jensen’s Inequality)  \hspace{0.5cm} [27] For any constant matrix $M \in R^{n \times n}$, $M^T > M > 0$, scalars $\alpha$ and $\beta$ with $\alpha > \beta$ and vector $x : [\beta, \alpha] \rightarrow R^n$, such that the following integrations are well defined, then

$$-(\alpha - \beta) \int_{\beta}^{\alpha} x^T(s)Mx(s) ds \leq - (\int_{\beta}^{\alpha} x(s) ds)^T M (\int_{\beta}^{\alpha} x(s) ds),$$

$$-\frac{(\alpha - \beta)^2}{2} \int_{\beta}^{\alpha} \int_{\beta}^{\alpha} x^T(s)Mx(s) ds du \leq - (\int_{\beta}^{\alpha} x(s) ds du)^T M (\int_{\beta}^{\alpha} x(s) ds du).$$

**III. MAIN RESULTS**

In this section, the stability criteria for neural networks with mixed time delay using reliable sampled data control is presented through the following theorem.

**Theorem 3.1.** Under Assumption 2.1, for given positive scalars $\rho$, $\tau$, $h$ then the neural network (5) is asymptotically stable if there exist positive definite matrices $P_1, Q_1, Q_2, Q_3, R_1, R_2, S_1, T_1, T_2$, any matrix $U$ with appropriate dimensions such that the following linear matrix inequality hold,

$$[\Omega]_{1x1} < 0, \hspace{1cm} (6)$$

where

$$\Omega_{1,1} = Q_1 + Q_2 + \tau^2 Q_3 + T_1 - T_2 + \frac{\tau^4}{4} S_1 - F_1 L, \Omega_{1,2} = P_1 - AU, \Omega_{1,3} = T_2 \Omega_{2,1} = F_2 L,$$

$$\Omega_{2,2} = h^2 T_2 - U - U^T, \Omega_{2,3} = G Y, \Omega_{2,4} = U B, \Omega_{2,5} = U C, \Omega_{2,6} = U D, \Omega_{2,7} = \frac{\tau^4}{4} S_1 - Q_1,$$

$$\Omega_{3,3} = F_2 S_1 \Omega_{3,4} = -2 T_3, \Omega_{3,5} = T_5 S_1, \Omega_{3,6} = -T_2, \Omega_{3,7} = -T_4 + \tau^2 R_2 - L,$$

and the values for other terms are zero. Moreover the control gain matrix is given by $K = U^{-1}Y$.

**Proof:** Consider the Lyapunov Krasovskii functional as follows:

$$V(x(t)) = \sum_{i=1}^{S} \int_{t-H_i}^{t} V_i(x(s)), \hspace{1cm} (7)$$

where

$$V_1(x(t)) = x^T(t)P_1 x(t), \hspace{0.5cm} V_2(x(t)) = \int_{t-\tau(t)}^{t} x^T(s)Q_1 x(s) ds + \int_{t-\tau(t)}^{t} x^T(s)Q_2 x(s) ds + \int_{t-\tau(t)}^{t} x^T(s)Q_3 x(s) ds du,$$

$$V_3(x(t)) = \int_{t-\tau(t)}^{t} \int_{t-\tau(t)}^{t} x^T(s)R_1 f(x(s)) ds + \int_{t-\tau(t)}^{t} \int_{t-\tau(t)}^{t} x^T(s)R_2 f(x(s)) ds du,$$

$$V_4(x(t)) = \frac{\tau^2}{2} \int_{t-H_i}^{t} \int_{t-H_i}^{t} x^T(s)S_1 x(s) ds du\theta,$$

$$V_5(x(t)) = \int_{t-H_i}^{t} \int_{t-H_i}^{t} x^T(s)T_1 x(s) ds + h \int_{t-H_i}^{t} \int_{t-H_i}^{t} x^T(s)T_2 x(s) ds du.$$
\[ -\tau \int_{t^{-\tau(t)}}^{t} x^T(s)Q_3x(s)ds \leq - \left( \int_{t^{-\tau(t)}}^{t} x^T(s) \right)^T Q_3 \left( \int_{t^{-\tau(t)}}^{t} x(s)ds \right), \]
\[ -\tau \int_{t^{-\tau(t)}}^{t} f(x(s))R_2f(x(s))ds \leq - \left( \int_{t^{-\tau(t)}}^{t} f(x(s))ds \right)^T R_2 \left( \int_{t^{-\tau(t)}}^{t} f(x(s))ds \right), \]
\[ -\frac{T}{2} \int_{t^{-\tau(t)}}^{t} x^T(s)S_1x(s)ds d\theta \leq - \left( \int_{t^{-\tau(t)}}^{t} x^T(s)ds \right)^T S_1 \left( \int_{t^{-\tau(t)}}^{t} x(s)ds d\theta \right). \]
Equation (12) can be written as
\[ -h \int_{t^{-\tau(h)}}^{t} x^T(s)T_2x(s)ds \leq -h \int_{t^{-\tau(h)}}^{t} x^T(s)T_2x(s)ds - h \int_{t^{-\tau(h)}}^{t} \hat{x}(s)T_2\hat{x}(s)ds. \]
By applying Jensen's inequality and simplifying it we get
\[ -h \int_{t^{-\tau(h)}}^{t} \hat{x}(s)T_2\hat{x}(s)ds \leq - \left( \int_{t^{-\tau(h)}}^{t} \hat{x}(s)ds \right)^T T_2 \left( \int_{t^{-\tau(h)}}^{t} \hat{x}(s)ds \right), \] where,
\[ \zeta(t) = \left[ x^T(t) x^T(t-\tau(t)) x^T(t-h(t)) x^T(t) f^T(x(t)) f^T(x(t-\tau(t))) \right] \]
\[ \int_{t^{-\tau(t)}}^{t} x^T(s)ds \int_{t^{-\tau(t)}}^{t} f^T(x(s))ds \int_{t^{-\tau(t)}}^{t} x^T(s)ds d\theta \]
From equation (6) we get, \[ V(\hat{x}(t)) < 0. \] Therefore by Definition 2.2 we get that the mixed delayed neural network is asymptotically stable. This completes the proof.

**Remark 3.2.** Now consider there is no distributed delay, then equation (5) becomes,
\[ \hat{x}(t) = -Ax(t) + B\hat{x}(t) + Cf(x(t-\tau(t))) + Gk\hat{x}(t-h(t)). \]
For the mixed delay neural networks (22), we can derive the stability conditions from Theorem 3.1, then we have the following Corollary 3.3.

**Corollary 3.3.** Under Assumption 2.1, for given positive scalars \( \rho, \tau, h \) the system (22) is asymptotically stable if there exist positive definite matrices \( P_1, Q_1, Q_2, Q_3, S_1, T_1, T_2 \), any matrix \( U \) with appropriate dimensions such that the following linear matrix inequality hold,
\[ \begin{bmatrix} \Omega_1^T & \Omega_2^T \end{bmatrix} < 0, \]
where,
\[ \Omega_1 = P_1 + \tau Q_2 + \frac{\tau^2}{4} Q_3 - F_1 L, \Omega_2 = P_1 - AU, \Omega_3 = \frac{1}{2} F_1 L \]
\[ \Omega_4 = \frac{1}{2} F_2 S, \Omega_5 = \frac{1}{2} F_2 S, \Omega_6 = \frac{1}{2} F_2 S, \Omega_7 = \frac{1}{2} F_2 S, \Omega_8 = \frac{1}{2} F_2 S, \Omega_9 = \frac{1}{2} F_2 S, \]
\[ \Omega_{10} = \frac{1}{2} F_2 S, \Omega_{11} = \frac{1}{2} F_2 S, \Omega_{12} = \frac{1}{2} F_2 S, \Omega_{13} = \frac{1}{2} F_2 S, \Omega_{14} = \frac{1}{2} F_2 S, \Omega_{15} = \frac{1}{2} F_2 S, \]
and the values for other terms are zero. Moreover the control gain matrix is given by \( K = U-U^T \).

**Proof:** Consider a Lyapunov Krassovskii functional,
\[ V_1(x(t)) = x^T(t)P_1 x(t), \]
\[ V_2(x(t)) = \int_{t^{-\tau(t)}}^{t} x^T(s)Q_1 x(s)ds + \int_{t^{-\tau(t)}}^{t} x^T(s)Q_2 x(s)ds + \int_{t^{-\tau(t)}}^{t} x^T(s)Q_3 x(s)ds d\theta, \]
\[ V_3(x(t)) = \int_{t^{-\tau(t)}}^{t} x^T(s)S_1 x(s)ds + \int_{t^{-\tau(t)}}^{t} x^T(s)S_2 x(s)ds + \int_{t^{-\tau(t)}}^{t} x^T(s)S_3 x(s)ds d\theta. \]
Using the same procedure as in Theorem 3.1, we get equation (23). This completes the proof.

**Remark 3.4.** There is no distributed delay and control, then equation (22) becomes,
\[ \hat{x}(t) = -Ax(t) + B\hat{x}(t) + Cf(x(t-\tau(t))), \]
For the mixed delay neural networks (24), we can derive the stability conditions from Theorem 3.1, then we have the following Corollary 3.5.

**Corollary 3.5.** Under Assumption 2.1, for given positive scalars \( \rho, \tau, h \) the system (24) is asymptotically stable if there exist positive definite matrices \( P_1, Q_1, Q_2, Q_3, S_1, T_1, T_2 \), any matrix \( U \) with appropriate dimensions such that the following linear matrix inequality hold,
\[ \begin{bmatrix} \Omega_1^T & \Omega_2^T \end{bmatrix} < 0, \]
where,
\[ \Omega_1 = P_1 + \tau Q_2 + \frac{\tau^2}{4} Q_3 - F_1 L, \Omega_2 = P_1 - AU, \]
\[ \Omega_3 = \frac{1}{2} F_1 L, \Omega_4 = \frac{1}{2} F_2 S, \Omega_5 = \frac{1}{2} F_2 S, \]
"
\[ \dot{x}_{6,8} = -S \hat{x}_{7,2} = -Q_3 \hat{x}_{8,0} = -S_1, \]
and the values for other terms are zero.

**Proof:** Consider a Lyapunov-Krasovskii functional as follows:
\[ V_1(x(t)) = x^T(t)P_1 x(t), \]
where \[ P_1 = \begin{bmatrix} 1.7 & 0 \\ 0 & -3 \end{bmatrix}, \]
\[ C = \begin{bmatrix} 1.5 & 0.2 \\ -0.6 & 0 \end{bmatrix}, \]
\[ D = \begin{bmatrix} 1.2 & 0.4 \\ 0 & -0.5 \end{bmatrix}. \]

Using the same procedure as in Theorem 3.1, we get equation (25). This completes the proof.

### IV. NUMERICAL EXAMPLES

In this section, a numerical example is provided to illustrate the efficacy of the theoretical results. Consider a mixed delay neural network as
\[
x(t) = -Ax(t) + Bf(x(t)) + Cf(x(t - \tau(t))) + D \int_{\tau(t)}^{t} f(x(s)) ds + G K x(t - h(t)),
\]
with the following parameter as
\[
A = \begin{bmatrix} -0.8 & 1 \\ 1.5 & -0.5 \end{bmatrix}, \quad B = \begin{bmatrix} 1.7 & 0 \\ 0 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} 1.5 & 0.2 \\ -0.6 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1.2 & 0.4 \\ 0 & -0.5 \end{bmatrix},
\]
and the matrices are
\[
F_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad G = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},
\]
and the remaining parameter are \( \tau = 1.5, \rho = 1, h = 0.1. \)

By solving the LMI in theorem 3.1 we get the solution as
\[
P_1 = \begin{bmatrix} 3.5858 & -3.3897 \\ -3.3897 & 4.1775 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} -13.5616 & 0.0219 \\ 0.0219 & -13.7823 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 2.8911 & -0.0065 \\ -0.0065 & 2.8986 \end{bmatrix},
\]
\[
Q_3 = \begin{bmatrix} 1.2623 & -0.0043 \\ -0.0043 & 1.2856 \end{bmatrix}, \quad R_1 = \begin{bmatrix} -30.6698 & 4.5041 \\ 4.5041 & -35.6705 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 18.7681 & -1.5775 \\ -1.5775 & 20.5269 \end{bmatrix},
\]
\[
S_1 = \begin{bmatrix} -0.0061 & 2.2771 \\ 2.2771 & 0.50185 \end{bmatrix}, \quad T_1 = \begin{bmatrix} 0.50185 & 0 \\ 0 & 0.50185 \end{bmatrix}, \quad S = \begin{bmatrix} -278.1110 & 0 \\ 0 & -278.1110 \end{bmatrix}, \quad U = \begin{bmatrix} 3.8007 & 0.1314 \\ 0.1314 & 2.4703 \end{bmatrix},
\]
\[
Y = \begin{bmatrix} -11.7099 & 0 \\ 0 & -0.05235 \end{bmatrix}, \quad -5.0235 & -5.7075 \end{bmatrix}.
\]

The control given matrix is given as \( K = \begin{bmatrix} -30793 & -0.0580 \\ -0.0481 & -2.3074 \end{bmatrix} \).

### V. CONCLUSION

In this paper, the reliable sampled data control problem for mixed delayed neural network is studied. By utilizing the integral inequality technique, and Lyapunov approach, a new set of conditions are obtained which ensures that the mixed delayed neural network is asymptotically stable for all possible actuator failures. Particularly, the reliable sampled data control law is designed in terms of the solution of certain linear matrix inequalities. The solvability of the concerned problem has been expressed as the feasibility of a set of LMI. Finally, a numerical example is given to validate the effectiveness of the proposed techniques.

### REFERENCES


