Development of Synthetic Unit Hydrograph for Ungauged Catchments – A Review

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Abstract

This paper primarily focusses on the development of synthetic unit hydrographs for ungauged catchments that are hydrologically homogeneous. The scope of this study is to review the efforts made by the researchers to develop suitable models for such catchments that can be used by planners for flood risk assessment and adoption of flood prediction and warning systems downstream. In this study, probabilistic distribution function-based models have been focused because of their good performance in previous applications to different catchments. We found that the probabilistic distribution function-based models that use a parametric approach and employ basin geomorphology and homogeneity to minimize the parameters offers a better approach. These models are robust, involve few parameters and provide good results for ungauged basins. Their main advantage is that once the regional homogeneity is established, standardized data from different sites within a region can be pooled together and a single frequency curve can be made applicable to the entire region.

Introduction

Unit hydrograph (UH) is a powerful tool in hydrology most widely used to assess flood risk and the adoption of flood prediction and warning systems for gauged basins. The accuracy of the prediction depends upon the sufficiency and accuracy of historical data. Sherman [1] was perhaps the first to evolve the procedure to develop the UH for a basin or watershed from its rainfall-runoff data. While developing the UH, Sherman assumed that the catchment is linear and time-invariant but failed to describe the runoff distribution precisely. This limits the application of UHs to gauged watersheds only. But the theory is being recognized as a good predictive tool and the UH is seen to reflect the characteristics of the basin it represents.

The basin characteristics that can be reflected in a UH are the drainage area, size and shape, drainage density and its distribution, length of the main stream, and storage due to the surface or channel obstructions. The theory of UH for gauged basins can be extended to hydrological prediction in ungauged catchments but require the construction of UH from the basin characteristics it represents and the observed rainfall-runoff data [2].

In this background, several studies have been made to synthesize a UH for ungauged basins. An excellent review of theories proposed for extension of UH to ungauged basins has been provided by Singh et al. [3]. However, in many parts of the world, rainfall and runoff data are seldom adequate to develop UH for a basin. The paucity of such data led to the concept of the
synthetic unit hydrograph (SUH). The term “synthetic” here denotes the hydrograph derived from basin characteristics rather than from measured rainfall-runoff data.

The synthetic unit hydrograph can be classified into four categories – traditional, conceptual, probabilistic, and geomorphological. The traditional SUH models are based on empirical equations and have certain region-specific constants/coefficients. These coefficients represent the physical characteristics of the catchments and vary over a wide range even within the same region. The notable traditional models include Snyder [4], Tailor and Schwarz [5] and Soil Conservation Service [6].

To determine the precise hydrologic response of both gauged and ungauged catchments, significant efforts have been made by the scientific community to reduce the number of such parameters. The probabilistic or probability distribution function-based models use a parametric approach and employ basin geomorphology and homogeneity to minimize the parameters and derive the hydrologic response of the catchments by developing different models. These models are robust, involve a few parameters and provide good results. Their main advantage is that once the regional homogeneity is established, standardized data from different sites within a region can be pooled together and a single frequency curve can be made applicable to the entire region.

This paper primarily focusses on the development of synthetic unit hydrographs for ungauged catchments that are hydrologically homogeneous. The scope of this study is to review the efforts made by the researchers to develop suitable models for such catchments that can be used for flood risk assessment and adoption of flood prediction and warning systems downstream. Probabilistic distribution function-based models have been focused in this study because of their good performance in previous applications to different catchments.

**Traditional Synthetic Unit Hydrograph Methods**

The beginning of the SUH concept can be traced back to the distribution graph proposed by Bernad [7] to synthesize SUH from watershed characteristics. Since then, numerous models have been developed. Some of the methods developed during the early stages are discussed below.

**Snyder Method**

In many parts of the world, rainfall and runoff data are seldom adequate to develop a unit hydrograph for a basin or watershed. Snyder [4] was perhaps the first to propose a method for the development of SUH by analyzing a larger number of basins in the Appalachian mountain region of the United States. To synthesize SUH Snyder evolved a set of empirical relations using watershed characteristics, i.e. the catchment area (A) (km$^2$); length of main stream (L) (km); the distance from the watershed outlet to a point on the main stream nearest to the center of the area of the watershed (Lc), and five characteristics the UH, i.e. the basin lag t_l, the peak direct runoff rate Q_p (m$^3$/s), the base time T_b (days), and the widths at 50 and 75 percent of the peak discharge, represented as W_50 and W_75 respectively.
These relationships can be expressed as:

\[ C_t = 0.3 \]  

(1)

\[ C_p = 2.78 \]  

(2)

where \( C_t \) is a regional coefficient ranging from 1.3 to 1.65 and \( C_p \) the storage coefficient accounting for retention and storage of rainfall in the catchment. For Indian watersheds, it varies from 0.56 to 0.69. The standard lag time \( t_l \) is measured in hours from the centre of the effective rainfall to the peak of the SUH and length of main stream \( L \) from the outlet to the farthest point on the catchment divide in Km. The standard duration of \( T_r \) (h) of UH (effective rainfall) adopted by Snyder is given by

\[ T_r = 5.5 \]  

(3)

If non-standard rainfall duration \( T_R \) is adopted, instead of the standard value \( T_r \), the basin lag is affected and needs revision. In such a case, modified basin lag \( t_l' \) is computed by

\[ t_l' = t_l + \frac{4}{T_R} \]  

(5)

The time base, \( T_b \) of the hydrograph is given by

\[ T_b = 3 + \frac{8}{T_R} \]  

(6)

To assist in the sketching the SUH, the widths of SUH at 50 and 75 percent of the peak are given by

\[ W_{50} = 5.87q_p \]  

(7)

\[ W_{75} = 50q_p / 1.75 \]  

(8)

where \( q_p = Q_p / A \).

The development of SUH involves the determination of five parameters, \( t_p, t_b, Q_p, W_{50}, \) and \( W_{75} \) using equations (1-8). Further, to have an area equal to unit depth of effective rainfall under the curve, manual adjustments are required which, however, is a tedious and cumbersome procedure and often involves errors.

**Taylor and Schwarz**

Taylor and Schwarz [5] derived empirical equations relating basin lag \( t_l \) and peak flow rate \( q_p \) to the catchment characteristics from the data collected from 20 catchments having varying drainage areas. They introduced the equivalent main stream slope as one of the important catchment characteristics affecting SUH. However, Taylor and Schwarz advocated the use of empirical equations for \( W_{50} \) and \( W_{75} \) to manually sketch the SUH.

**SCS Method**

Soil Conservation Service (SCS) [6] adopted a dimensionless hydrograph approach (i.e. \( q/q_p \) versus \( t/t_l \)) to develop SUH for ungauged catchments. Expressions relating time to peak \( t_p \) with peak discharge \( q_p \) as a function of catchment characteristics and curve numbers were proposed to develop SUH from the specified dimensionless UH.

**Statistical Distribution Method**

The statistical or probabilistic distribution function-based models use a parametric approach and employ basin geomorphology and homogeneity to minimize the parameters and derive the hydrologic response of the
catchments by developing different models. These models are robust, involve few parameters and provide good results for ungauged basins. Their main advantage is that once the regional homogeneity is established, standardized data from different sites within a region can be pooled together and a single frequency curve can be made applicable to the entire region.

Two-parameter Gamma, three-parameter Beta, two-parameter Weibull, and one-parameter Chi-square distribution are the popular choices among the researchers. Excellent stability analysis of these distributions has been provided by P. K. Bhunya et al [8].

**Gamma Distribution**

The Gamma distribution is based on the concept of the Nash [9] model and has been attempted by many authors. It comprises of \( n \)-equal linear reservoirs with equal storage coefficient \( K \). Nash [9] and Dooge [10] used the Gamma distribution to represent the instantaneous unit hydrograph (IUH) in the form

\[
q(t) = \frac{1}{\Gamma(n)} \left( \frac{t}{n} \right)^{n-1} e^{-\frac{t}{n}}
\]

in which \( q(t) \) is the depth of direct runoff per unit time per unit effective rainfall; \( \Gamma(n) \) is the Gamma function. The Nash model parameters \( n \) and \( K \) define the shape of the distribution; \( n \) being dimensionless and \( K \) has the unit of time. The mean and variance of the distribution (9) are given by

\[
\mu = \frac{n}{\Gamma(n)} \quad \sigma^2 = \frac{n}{\Gamma(n)^2}
\]

Equation (9) can be used to derive SUH from known values of \( n \), \( K \) and \( \Gamma(n) \). It no longer involves the computation of \( T_b \), \( W_{50} \), and \( W_{75} \). However, the parameters \( n \) and \( K \) need to be determined in terms of the geomorphologic parameters of the watershed. Chow [11] related the two parameters \( n \) and \( K \) using the relation

\[
\mu = \frac{n}{\Gamma(n)} = \frac{n}{K}
\]
Defining a non-dimensional shape parameter as the product of $q_p$ combining equations (9) and (11), the β can be written as

\[ \beta = q_p \text{ and } t_p, \text{ i.e. } \]

(12)

There is no exact solution of (12) to determine the value of β in terms of n and Gamma function. Attempts have been made by several authors [12, 13, 14, 15, 16, 17] to develop robust hydrological models to estimate these parameters from the catchment characteristics.

Croley [13] proposed a method to compute the value of n from the known values of peak runoff depth $q_0$ and the time to peak $t_l$ which is iterative and takes a lot of time to converge. McCuen [15] proposed an empirical relation to computing n represented as

\[ n = 0.045 + 0.5 \alpha + 5.6 \beta^2 + 0.3 \beta^3 \]

(13)

in which f is the depth of runoff per unit time per unit effective rainfall computed from

\[ f = \text{ [Value]} \]

(14)

Michel [16] expressed (12) in a quadratic form as

\[ n = 1 + \sqrt{1 + \alpha^2 \beta^2} \]

(15)

A relatively simple method is provided by S. K. Singh [17] to compute n using the empirical equation

\[ n = 1.09 + 0.164 \beta + 6.19 \beta^2 \]

(16)

Singh observed that the error in computing n using (16), decreases as the value of β increases.

S K Singh [18] proposed a method for transmuting SUHs, developed by Snyder, SCS, and Gray, into simplified dimensionless Gamma distribution which has only two parameters (i.e., shape parameter β and scale parameter $t_l$.) The calculations can be easily performed in a spreadsheet or on a calculator. It provides a smooth shape to a SUH and the area under the curve is guaranteed to be unity. It was proposed that by using (8), the ratio of runoff at any time $t$ to peak runoff, (i.e. $q/q_p$ ) can be written as

\[ \frac{q}{q_p} = \left( \frac{\alpha}{\alpha_0} \right) \]

(17)

where $q_p$ is the peak runoff per unit depth per unit effective rainfall, α is a function of $t/t_p$ given by

\[ \alpha = \left( \frac{t}{t_p} \right) + \left( \frac{t}{t_p} \right) \]

(18)

By approximating Gamma function by a suitable trigonometric function and inverting (12), an analytical expression for n, can be written as

\[ n = \frac{7}{2} + 2 \beta^2 \]

(19)
In equation (19), \( n \) depends only on the catchment characteristics and does not include the storage property of the catchment. It gives slightly higher values of \( n \) than its true value, but the error decreases with the increase in the value of \( \beta \). Using (16), (17) and (19), the ordinates of the SUH at time \( t \) can be computed from

\[
\left( \frac{1}{\beta} \right) \left[ \frac{(1 + 2 \beta)^{1/2}}{(1 + 2 \beta^{1/2})} \right] \tag{20}
\]

Eq. (20) represents a dimensionless form of Gamma distribution that contains two parameters (i.e., shape parameter \( \beta \) and scale parameter \( t_p \)). It also involves the determination of three parameters i.e. \( q_p \), \( t_p \), and \( \beta \). Knowing their values, the ordinates of a SUH can be obtained from (20). No manual adjustments are required and the area under the curve (20) is unity. It is a simple method and involves only the computation of storage coefficient \( C_p \) which is a function of catchment characteristics and needs calibration. The coefficient may vary significantly from catchment to catchment. To develop SUH for ungauged catchments, Linsley et al. [19] recommended that \( C_p \) would first be computed for catchments in the vicinity of the ungauged catchments.

However, the representation of \( q_p \) and \( t_p \) in terms of shape parameter \( \beta = t_p q_p \) eliminates the need of computation of \( C_p \). Knowing the value of \( \beta \), it can be calculated using (2) by expressing it in a dimensionally homogenous form and subsuming \( t_l \) from (4) as

\[
\beta = \left( \frac{1}{t + 2} \right) \tag{22}
\]

Further, using (3) gives (22)

Since \( \beta \) is a function of only catchment characteristics [20], it can be computed from (19), knowing the value of \( n \) from other sources. The computation, however, involves some error which decreases with the increase in the value of \( \beta \). Its value has shown to vary from 0.2 to 1.25 [18] with the minimum being of the order of 0.35. Thus, knowing \( t_r \), \( t_l \) and selecting an appropriate value of \( \beta \), the use of equation (20) facilitates the construction of SUH for ungauged catchments.

**Results and Discussions**

It has been observed that traditional methods based on Snyder Method, Taylor and Schwarz and SCS are still being used widely for planning and flood warning systems despite their inherent limitations and cumbersome procedure. The statistical or probabilistic distribution function-based models use a parametric approach and employ basin geomorphology and homogeneity to minimize the parameters and derive the hydrologic response of the catchments by developing different models. These models are robust, involve few parameters and provide good results for ungauged basins. Their main advantage is that once the regional homogeneity is established, standardized data from different sites within a region can be pooled together and a single frequency curve can be made applicable to the entire region.

Two-parameter Gamma, three-parameter Beta, two-parameter Weibull, and one-parameter Chi-square distribution are the popular choices among the researchers. The Gamma distribution based on the concept of the Nash model offers a convenient choice.

**References**


