Dynamics of Bianchi universe-I cosmological models in the existence of general relativity

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Abstract:
In this paper we explore, a model in the existence of general relativity for the context of Bianchi form-I cosmological models. For the sake of discovering a deterministic model of the cosmos and to get the particular solutions of the field equations of Einstein, we suppose Hubble parameter which gives the fixed value of deceleration parameter that acquires the required results of the field equations of Einstein. This parameter in the model is observed to be time subordinate which shows the development from initial decelerating period to the present accelerating period of expansion and supplies the biggest value and the fastest rate at which the universe is expanding. Model offers dynamically expansion to the universe which allows for symmetrical Bianchi metrics. The field equations of Einstein are observed in this solution, used for space periods with a perfect fluid or electromagnetic field as an origin. For the pressure-free matter, results of field equations of Einstein that displays the local rotational symmetry. A categorization of these results for a perfect fluid is provided, and supposing a physically reasonable equation of state, certain required results of cosmological significance are acquired and the same general classification can be made.

Keywords: Cosmological parameters, Cosmology, Field Equations.

I. INTRODUCTION
Cosmology is analysis of the fundamental systems and movements of our cosmos which concerns with their foundation and growth. It also includes learning about the movements of the heavenly bodies. The universe is not only isotropic but also uniform according to the current condition of growth. In cosmology, the problem of cosmological constant is considered to be one of the greatest problem. In the Einstein’s field equations, this cosmological constant problem was primarily specified. This can be done by considering different values for cosmological constant. In recent years, there has been significant and essential evidence about the recognition of Lambda as an Einstein's cosmological constant. A theory of universe combines dynamic degrees of freedom with matter fields. Therefore through development of cosmos and formation of elements, (Λ) reduces to its current slight value as the universe is very old so this constant value is very small. In addition the estimated worth of the cosmological concept and the expected value of the Quantum Field Theory (QFT) of the standard model is one of the outstanding problems in Physics. Many authors [1-10] have gone through the problem of cosmological term and after many attempts they have suggested that it takes on dynamical attributes and some of the followed the ideal of decreasing vacuum energy density with cosmic expansion, in order to understand the incredible of little estimation of the term-Λ, from its Quantum field theory calculated value which is recently observed, time dependent can be helpful to understand development of Λ.

With regard to such a vast entropy per baryon, physical phenomenon, as perceived, the noticeable degree of radiation of cosmic background in cosmology, indicates dissipative effect. As it is observed today, for a long time, the process of dissipation continued, and in the early era of cosmic development, many causes the high degree of isotropy. There are two main elements, the dissipative effect and the coefficient of bulk and shear viscosity, that are supposed to be particularly significant in the creation and foundation of the universe. On the account of this assumption, Weinberg [11] estimates and applies bulk and shear viscosity formulations that determine the role of cosmological entropy production because of the basis of this assumption. Padmanabhan and Chitre [12] noticed the existence of bulk viscosity with reference to general relativity, which causes solutions to be formed such as inflation etc. Ries[13] observed a basic description of the bulk viscosity which behaves as a real representative in the growing universe, that is called the theory of the negative entropy field. Referencing the viscosity dissipation method, Bilinski and Khalatnikov[14] considered the existence of the cosmological solutions for the homogeneous Bianchi type-I system. The Bianchi type I solution with shear viscosity being the power function of entropy density, in the case of stiff matter, explored by Banerjee et al [15]. Referencing the cosmological development, Hunag [16], Bali[17-19] and Baghel [20-25], Peebles [26] and Singh et. al. [27] revealed the outcome of bulk viscosity.

Cosmological concepts and scientific application of the outcome of FRW models encourages the concept of uniformity and isotropy of the universe. According to the viewpoint of above symmetries, the universe is neither uniform nor isotropic. The
existence of anisotropic phase supported by many theoretical opinions Lima [28], Lornez [29] and recent experimental data regarding cosmic background radiation anisotropies which approaches an isotropic discussed by Lukash[30]. The FRW model plays a very important role not only physically but also geometrically in the dissipation of the initial universe because of the cosmological models that are spatially uniform and anisotropic with the abundant formation. Flat FRW models were determined as anisotropic and exciting for further investigation just like Bianchi Type I models. These models supported by the facts from study of low density universe. Mark and Harko [31] has reflected the subtleties of a casual bulk viscous fluid cosmological model with continuously slowing cosmological models of Bianchi form I. Saha [32-33] considered with viscous fluid of the cosmological models of Bianchi form I. By using Law of variation of Hubble’s parameter, Einstein’s field equations can be solved which gives deceleration parameter is constant suggested by Berman [34]. Berman [34-35], Desikan [36],Singh and Desikan [37], Maharaj and Naidoo [38], Pradhan and et al [39] has considered the previous literature, a fixed deceleration parameter in cosmological models. There is cheque history for red shift magnitude test. It was used to draw very unconditional decisions for the duration of the 1960 and 1970. It was declared that deceleration parameter stated to lie between 0 and 1 and the universe is decelerating. Pradhan and et al [39-41] observed type Ia supernovae, and concluded that the extension of the universe is increasing and the cosmological models with variable deceleration parameter has been also examined by them. Vishwakarma and Narlikar[42] ande Virely et al [43] also considered the cosmological models with variable deceleration parameter.

With bulk viscous and time dependent deceleration parameter, cosmological model of Bianchi form - I has been discussed in this paper and it is assumed as one of the linear functions of Hubble’s parameter including deceleration parameter, adding significance of scale factor, we have obtained the required solution. The Prime purpose of proposed work is to explored lambda in the model of the perfect fluid which contains matter in the homogeneous Bianchi form I, for the stiff matter. The manuscript leads by the series of sections as mentioned: In section 2, the basic definitions of anisotropic models are mentioned. In the division 3, field equation’s results are obtained by proposing the cosmological term which is proportionate to the Hubble parameter square.In section 4, the conclusion drawn from the results.

II. METRIC AND FIELD EQUATIONS

Equation of the Bianchi form - I space time is given by

$$ds^2 = -dt^2 + R_1^2(t) dx^2 + R_2^2(t) dy^2 + R_3^2(t) dz^2$$

(1)

here the $R_1, R_2, R_3$ are metric potentials, that denotes cosmic time functions $t$.

Consider \( T_{ij} = (\rho + \overline{p})v_i v_j + \overline{p}g_{ij} \)

(2)

\( \overline{p} \) is signifying the effective pressure by

$$\overline{p} = p - \xi v_{ij}$$

(3)

$$p = \omega \rho$$

(4)

Matter density is represented by $\rho$, isotropic pressure is denoting by $p$, $\eta$ is coefficients of shear viscosity, $\xi$ is bulk viscosity and the flow vector of the fluid is represented by $v_i$ holds for

$$v^i v_i = -1.$$

The shear tensor $\sigma_{ij}$, expansion scalar $\theta$ and shear scalar $\sigma$ are given by:

$$\theta = v^i_i$$

(5)

$$\sigma_{ij} = \frac{1}{2} (v_i, k h^k_j + v_j, k h^k_i) - \frac{1}{3} \theta h_{ij}$$

(6)

$$\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij}$$

(7)
The projection is
\[ h_{ij} = g_{ij} + v_i v_j \]

With time differing cosmological term lambda Einstein's field conditions is described as:
\[ R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} + \Lambda g_{ij} \] (8)

By using condition (8) in the line component (1)
\[ \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} = -p + (\xi)\theta + \Lambda \] (9)
\[ \frac{\dot{R}_3}{R_3} + \frac{\dot{R}_1}{R_1} + \frac{\dot{R}_3 \dot{R}_1}{R_3 R_1} = -p + (\xi)\theta + \Lambda \] (10)
\[ \frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} = -p + (\xi)\theta + \Lambda \] (11)
\[ \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} + \frac{\dot{R}_3 \dot{R}_1}{R_3 R_1} = \rho + \Lambda \] (12)

“a” is represented as a normal scale factor
\[ a^3 = ABC \] (13)

Hubble-parameter is denoted by ‘H’ and deceleration parameter is denoted by q are characterized by
\[ H = \frac{\dot{a}}{a} = \frac{1}{3} (H_1 + H_2 + H_3) \] (14)
\[ q = -\frac{a\ddot{a}}{a^2} = \frac{d}{dt} \left( \frac{1}{H} \right) - 1 \] (15)

The Directional Hubble components along with the directions x, y & z respectively are
\[ H_1 = \frac{\dot{R}_1}{R_1} \]
\[ H_2 = \frac{\dot{R}_2}{R_2} \]
\[ H_3 = \frac{\dot{R}_3}{R_3} \]

‘\sigma_{ij}’ is represented by shear tensor, the expansion scalar is \( \theta \) then the equation (1) reduces to
\[ \theta = 3H = 3 \frac{\dot{a}}{a} \] (16)
\[ \sigma_{11} = H_1 - H \]
\[ \sigma_{22} = H_2 - H \]
\[ \sigma_{33} = H_3 - H \]
\[ \sigma_{44} = 0 \] (17)
\( \sigma \) is shear scalar
\[ \sigma^2 = \frac{1}{6} \left\{ \left( \frac{\dot{R}_1}{R_1} - \frac{\dot{R}_2}{R_2} \right)^2 + \left( \frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3} \right)^2 + \left( \frac{\dot{R}_3}{R_3} - \frac{\dot{R}_1}{R_1} \right)^2 \right\} \] (18)
By taking (9)-(11)
\[
\frac{\dot{R}_1}{R_1} - \frac{\dot{R}_2}{R_2} + \left(\frac{\dot{R}_1}{R_1} - \frac{\dot{R}_2}{R_2}\right)\left(\frac{\dot{R}_3}{R_3}\right) = 0
\]  
(19)
\[
\frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3} + \left(\frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3}\right)\left(\frac{\dot{R}_1}{R_1}\right) = 0
\]  
(20)

By doing the integration, the equations (19) and (20),
\[
\frac{\dot{R}_1}{R_1} - \frac{\dot{R}_2}{R_2} = k_1 \quad a^3
\]  
(21)
\[
\frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3} = k_2 \quad a^3
\]  
(22)

Integration constants are \(k_1\) and \(k_2\)

By taking equations (14), (21)-(22)
\[
\frac{\dot{R}_1}{R_1} = \frac{\dot{a}}{a} + \frac{k_2+2k_1}{3a^3}
\]  
(23)
\[
\frac{\dot{R}_2}{R_2} = \frac{\dot{a}}{a} + \frac{k_2-k_1}{3a^3}
\]  
(24)
\[
\frac{\dot{R}_3}{R_3} = \frac{\dot{a}}{a} - \frac{2k_2+k_1}{3a^3}
\]  
(25)

Consider \(\xi = \xi_0 + \xi_1 H\)

(26)

where \(\eta_0\), \(\xi_0\) & \(\xi_1\) are the constants.

Equations (23)-(25) and (26) reduce to
\[
\frac{\dot{R}_1}{R_1} = \frac{\dot{a}}{a} + \frac{k_2+2k_1}{3a^3}
\]  
(27)
\[
\frac{\dot{R}_2}{R_2} = \frac{\dot{a}}{a} + \frac{k_2-k_1}{3a^3}
\]  
(28)
\[
\frac{\dot{R}_3}{R_3} = \frac{\dot{a}}{a} - \frac{2k_2+k_1}{3a^3}
\]  
(29)

We obtain
\[
\sigma_{11} = \frac{k_2+2k_1}{3a^3}
\]  
(30)
\[
\sigma_{22} = \frac{k_2-k_1}{3a^3}
\]  
(31)
\[
\sigma_{33} = -\frac{2k_2+k_1}{3a^3}
\]  
(32)
\[
\sigma_{44} = 0
\]  
(33)

Therefore \(\sigma = \frac{k_2}{a^3}\)

(34)

where \(3k_2^2 = k_1^2 + k_2^2 + k_1k_2\)

(35)

Writing the equations (9)-(12) in the form of \(H, \sigma\) and \(q\)
\[
\bar{p} - \Lambda = (2q - 1)H^2 - \sigma^2
\]  
(36)
\[ \rho + \Lambda = 3H^2 - \sigma^2 \]  

(37)

From equations (36)-(37) we get

\[ \dot{H} = -3H^2 + \frac{1}{2} (\rho - p) + \frac{3}{2} \xi H + \Lambda \]  

(38)

**III. SOLUTION OF THE FIELD EQUATIONS**

By assuming \( q \) as the deceleration parameter, \( H \) is Hubble’s parameter is

\[ q = -1 + \beta H \]  

(39)

which consequents

\[ a = e^{\frac{1}{2} \beta t + k} \]  

(40)

where \( \beta \) and \( k \) are the constants.

From equations (14)-(16),(26) and (34), we get:

\[ H = \frac{1}{\sqrt{2 \beta t + k}} \]  

(41)

\[ \theta = \frac{3}{\sqrt{2 \beta t + k}} \]  

(42)

\[ \sigma = \frac{k_3}{e^{\frac{1}{2} \beta t + k}} \]  

(43)

\[ q = \frac{\beta}{\sqrt{2 \beta t + k}} - 1 \]  

(44)

The anisotropy parameter \( \bar{A} \) for the model is given by

\[ \bar{A} = \frac{2 \sigma^2}{3H^2} = \frac{2 k_3^2 (2\beta t + k)}{9 e^{\frac{1}{2} \beta t + k}} \]  

(45)

\( \eta \) is the coefficient of shear viscosity, \( \xi \) is the bulk viscosity

\[ \eta = \frac{3 \eta_0}{\sqrt{2 \beta t + k}} \]  

(46)

\[ \xi = \xi_0 + \frac{\xi_1}{\sqrt{2 \beta t + k}} \]  

(47)

Cosmological term is denoted by \( \Lambda \) and the matter density is denoted by \( \rho \) are given as:

\[ \rho = \frac{1}{(1+\omega)} \left( \frac{2\beta}{(2\beta t+k)^3} - \frac{2k_3^3}{e^{\frac{1}{2} \beta t + k} (2\beta t+k)^3} + \frac{3}{\sqrt{2 \beta t + k}} \left( \xi_0 + \frac{\xi_1}{\sqrt{2 \beta t + k}} \right) \right) \]  

(48)

\[ \Lambda = \frac{3}{2\sqrt{2 \beta t + k}} - \frac{2\beta}{(1+\omega)(2\beta t+k)^{3}} + \frac{(1-\omega)}{(1+\omega)} \left( \frac{2\beta}{(2\beta t+k)^3} - \frac{2k_3^3}{e^{\frac{1}{2} \beta t + k} (2\beta t+k)^3} + \frac{3}{\sqrt{2 \beta t + k}} \left( \xi_0 + \frac{\xi_1}{\sqrt{2 \beta t + k}} \right) \right) \]  

(49)
In the given model value of $t$ is specified by

$$t = -\frac{k}{2\beta} = t' \ (k \geq 0, \ \beta > 0)$$

which can be shifted to $t = 0$ by setting $k_3 = 0$.

Here, the radius scale factor is denoted by $a$ which is constant that is in the given model that is the universe is free from initial singularity. Also $H$ is denoted by the Hubble parameter, $\theta$ is the expansion scalar, $p$ is isotropic pressure and $\rho$ is energy density, are infinite at time

$$t = t'$$

whereas the shear scalar $\sigma$ is constant.

Now as $t$ increases scale factor $a$ also increases while the parameters $H, \theta, p, \rho, \sigma$ all decrease and by taking the large value of $t$, scale factor $a$ becomes extremely huge but all the parameters $H, \theta, p, \rho, \sigma$ approaches to 0. At the early time $t = t'$, the cosmological density lambda is infinite and $\Lambda \to 0$ as $t \to \infty$. In the present model, for the $t < \frac{\beta^2 - k}{2\beta}$, the deceleration parameter $q$ is positive that shows the decelerating phase of development and when $t > \frac{\beta^2 - k}{2\beta}$, the $q$ is negative that calculates the accelerating phase of extension of the universe. However, as $t \to \infty$, $q = -1$.

From equation (45), we see that the anisotropy parameter $\overline{A}$ converges to 0 as $t$ converges to infinite. Therefore the model approaches isotropy for large $t$.

**IV. CONCLUSION**

In this investigation, uniform and exclusively anisotropic Bianchi type-I space time are considered. We examined a cosmological scenario assuming a variation law in which deceleration parameter $q$ that is

$$q = -1 + \beta H$$

that gives the scale factor as $a = e^{\frac{1}{\beta^2}t + k}$ that is simple linear function of Hubble’s parameter $H$. We have obtained the cosmological models in which the Universe grows exponentially with time $t$ and begins from a non-singular state. The deceleration parameter $q$ in the model is found to be time dependent. A transition from the initial deceleration stage to the new accelerated expansion process is indicated by $q$, and it supplies the largest value and the accelerated rate at which the universe is expanding. The cosmological term $\Lambda$ approaches to zero because of $t$ approaches to infinite, which is also shown by current interpretations.
REFERENCES


