Numerical Study of a Micropolar Fluid past a Stretched Permeable Surface under Induced Magnetic Field

[1]Smita Sahu, [2]Dr.G.C.Hazarika

[1] Assistant Professor, Department of Mathematics, Dibru college, Dibrugarh, Assam, India
[2] Professor, Department of Mathematics, Dibrugarh University, Dibrugarh, Assam, India

Abstract : Numerical study of a micropolar fluid past a stretched permeable surface under induced magnetic field with variable viscosity and thermal conductivity have been studied. Viscosity and thermal conductivity are taken to be the inverse linear functions of temperature. To find the numerical solutions using Runge-Kutta fourth order method with shooting technique, the governing equations which are nonlinear, are reduced to dimensionless forms using similarity transformations. The effects of viscosity parameter, thermal conductivity parameter and the other parameters involved in the study on velocity, micro-rotation, temperature and induced magnetic field distribution profiles are investigated. The results are shown graphically and in tabulated form and discussed in details.

***Keywords* – Induced magnetic field, Micropolar fluid, shooting technique, thermal conductivity, viscosity.**

I. INTRODUCTION

The problems of micropolar fluids past a stretching surface has been the field of very active research due to their numerous technological applications. Micropolar fluids are defined as the fluids that contain micro-constituents that can undergo rotation which affect the hydrodynamics of the flow which are distinctly non-Newtonian in nature. These fluids has a microstructure and exhibit microrotational effects and can support surface and body couples which are not present in the theory of Newtonian fluids. Initially, Eringen [1] developed the theory of microfluids which include the effect of local rotary inertia, the couple stress and inertia spin. Eringen [2] also developed the theory of micropolar fluids for the case where only microrotational effects and microrotational inertia exists and later on he [3] extended the theory of thermomicropolar fluids and derived the constitutive law for fluids with microstructure. An review of micropolar fluids and their applications has been given by Ariman *et al.* [4]. Micropolar fluids can be defined in view of Lukaszewicz [5] as the fluids which consist of randomly oriented particles suspended in a viscous medium. Gorla [6] investigated the forced convective heat transfer of a micropolar fluid flow over a flat plate. The boundary layer flow of a micropolar fluid past a semi -infinite plate was studied by Ahmadi [7] taking into account the gyration vector normal to the xy-plate and the microinertia effects. The effects of variable viscosity and thermal conductivity on non-Newtonian micropolar fluid flow with heat generation were studied by Borgohain *et al.* [8]. Borthakur *et al.* [9] investigated the effects of variable viscosity and thermal conductivity on flow and heat transfer over an unsteady stretching sheet in a micropolar fluid with prescribed surface heat flux. Ali *et al*. [10] discussed the MHD mixed convection boundary layer flow under the effect of induced magnetic field. The effects of variable viscosity and thermal conductivity on MHD flow of a micropolar fluid in a continuous moving flat plate was discussed by Phukan *et al.* [11]. MHD flow of a micropolar fluid past a stretched permeable surface with heat generation or absorption was investigated by Khedr *et al.* [12].
In most of the studies mentioned above, the physical properties were considered as constants. But these properties may vary with temperature changes. The flow patterns are significantly changed in comparison of constant property case when the effects of variable viscosity and thermal conductivity are taken into account. In this paper, numerical study of the effects of variable viscosity and thermal conductivity of a MHD micropolar fluid past a stretched semi-infinite vertical plate under induced magnetic field is discussed. By Lai and Kulacki [13], the fluid viscosity and thermal conductivity are taken as inverse linear functions of temperature. Using similarity transformations the governing partial differential equations of motion are reduced to ordinary differential equations and are solved numerically by shooting technique.

II. MATHEMATICAL FORMULATION

We consider the steady two dimensional flow of a viscous incompressible MHD micropolar fluid past a permeable uniformly stretched semi-infinite vertical plate in presence of heat generation or absorption, thermal radiation and viscous dissipation effects. The magnetic Reynolds number of the flow is taken to be large enough so that the induced magnetic field is not negligible. A uniform induced magnetic field of strength H_0 is assumed to be applied in the positive y- direction normal to the vertical plate. The normal component of the induced magnetic field H_y vanishes when it reaches the wall and the parallel component H_x approaches the value of H_0.
Under these assumptions, along with the boundary-layer approximations, the governing equations are as follows:

Continuity Equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

Gauss Law of magnetism:

$$\frac{\partial H_x}{\partial x}+\frac{\partial H_y}{\partial y}=0 \tag{2}$$

Momentum Equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})+\frac{k}{\rho}(\frac{\partial N}{\partial y}+\frac{\partial^2 u}{\partial y^2})+\frac{\mu_e}{4\pi\rho}(H_x\frac{\partial H_x}{\partial x}+H_y\frac{\partial H_x}{\partial y}) \tag{3}$$

Angular momentum Equation:

$$\frac{\gamma}{k}\frac{\partial^2 N}{\partial y^2}-2N-\frac{\partial u}{\partial y}=0 \tag{4}$$

Magnetic induction Equation:

$$u\frac{\partial H_x}{\partial x}+v\frac{\partial H_x}{\partial y}=H_x\frac{\partial u}{\partial x}+H_y\frac{\partial u}{\partial y}+\eta_e\frac{\partial^2 H_x}{\partial y^2} \tag{5}$$

Energy Equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}(\lambda\frac{\partial T}{\partial y})+\frac{1}{\rho c_p}(\mu+k)\left(\frac{\partial u}{\partial y}\right)^2+\frac{1}{\rho c_p}Q(x)(T-T_\infty)-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{6}$$

Where u and v are the components of velocity along x and y- directions respectively, ρ is the fluid density, μ is the coefficient of dynamic viscosity, k is the vortex viscosity, N is the microrotation component, γ is the spin gradient viscosity, T is the temperature of the fluid, λ is the thermal conductivity , c_p is the specific heat at the constant pressure, Q(x) is the heat generation (>0) or absorption (<0) coefficient and q_r *is* the radiative heat flux.
The boundary conditions are given as:

$$y=0: u=U_0,\ v=v_w, N=0, \frac{\partial H_x}{\partial y}=H_y=0, T=T_w$$

$$y\to\infty: u\to 0,\ N\to 0, H_x=H_0, T\to T_\infty \tag{7}$$

Where U_0, v_w *and* T_w are the stretching velocity, suction (v_w<0) or injection (v_w>0) velocity and wall temperature respectively and T_∞ is the temperature of the fluid at infinity. By using Rosseland approximation and following El-Arabawy [14], the radiative heat flux q_r is given by

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial(T^4)}{\partial y}$$

Where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. By taking the temperature differences are sufficiently small within the flow and expanding T^4 usingTaylor series about the free stream temperature T_∞ such that

$$T^4=4T_\infty^3 T-3T_\infty^4$$

where the higher-order terms of the expansion are neglected.
Following Lai and Kulacki ,

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}[1+\delta(T-T_\infty)] \text{ or } \frac{1}{\mu}=\zeta(T-T_r) \text{ where } \zeta=\frac{\delta}{\mu_\infty} \text{ and } T_r=T_\infty-\frac{1}{\delta}$$

$$\frac{1}{\lambda}=\frac{1}{\lambda_\infty}[1+\xi(T-T_\infty)] \text{ or } \frac{1}{\lambda}=\varepsilon(T-T_c) \text{ where } \varepsilon=\frac{\xi}{\lambda_\infty} \text{ and } T_c=T_\infty-\frac{1}{\xi} \tag{8}$$

Where μ_∞ is the viscosity at infinity, ζ and T_∞ are constants, T_r is transformed reference temperature , δ and ξ are constants based on thermal property of the fluid. Similarly, λ_∞ is the thermal conductivity at the infinity, ε and T_c are constants and their values depend on the reference state and thermal properties of the fluid.
To solve equations (1)-(6) along with the boundary conditions given in equation (7) we use the following similarity transformations,

$$\eta=(\frac{U_0}{2\upsilon_\infty x})^{\frac{1}{2}}y, \psi=(2\upsilon_\infty U_0 x)^{\frac{1}{2}}f(\eta),\ \psi=(2\upsilon_\infty U_0 x)^{\frac{1}{2}}f(\eta),\quad N=U_0(\frac{U_0}{2\upsilon_\infty x})^{\frac{1}{2}}g(\eta),$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\quad u=U_0 f'(\eta), v=-\sqrt{\frac{\upsilon_\infty U_0}{2x}}[f(\eta)-\eta f'(\eta)]$$

$$H_x=H_0 H'(\eta),\ H_y=-\sqrt{\frac{\upsilon_\infty H_0}{2x}}[H(\eta)-\eta H'(\eta)]$$

$$u=\frac{\partial \psi}{\partial y}, v=-\frac{\partial \psi}{\partial x} \tag{9}$$

where η is the similarity parameter and υ_∞ is the kinematic viscosity at $T=T_\infty$.

Also from equations (7) and (8),we have,

$$\upsilon=-\upsilon_\infty\frac{\theta_r}{\theta-\theta_r}, \lambda=-\lambda_\infty\frac{\theta_c}{\theta-\theta_c} \tag{10}$$

where θ_r and θ_c are the dimensionless parameters characterising the influence of viscosity and thermal conductivity respectively and are given by,

$$\theta_r=\frac{T_r-T_\infty}{T_w-T_\infty}-\frac{1}{\delta(T_w-T_\infty)},$$
$$\theta_c=\frac{T_c-T_\infty}{T_w-T_\infty}=-\frac{1}{\xi(T_w-T_\infty)} \tag{11}$$

Equations (1) and (2) are identically satisfied using equation (9) and therefore the velocity field and magnetic fields are compatible and represent the possible fluid motion.

Using equations (8) - (11) in equations (3)-(6) the following differential equations are obtained:

$$\left(\frac{\theta_r}{\theta-\theta_r}-K\right)f'''=\left\{\frac{\theta_r}{(\theta-\theta_r)^2}\theta'+f\right\}f''+Kg'-MHH'' \tag{12}$$

$$g''=\frac{1}{G}\left(4g+2f''\right) \tag{13}$$

$$H'''=\frac{1}{\beta}(Hf''-fH'') \tag{14}$$

$$\left(3Nr\frac{\theta_c}{\theta-\theta_c}-4\right)\theta''=3Nr\Pr f\theta'+\left\{3Nr\frac{\theta_c}{(\theta-\theta_c)^2}\right\}(\theta')^2+3Nr\Pr Ec\left(K-\frac{\theta_r}{\theta-\theta_r}\right)(f'')^2+3Nr\Pr\phi\theta \tag{15}$$

Where the primes denote differentiation with respect to η and

$K=\frac{k}{\mu_\infty}$ is the coupling constant parameter ,

$M=\frac{\mu_e H_0^2}{4\pi\rho U_0^{\,2}}$ is the induced magnetic parameter ,

$G=\frac{\gamma U_0}{k\upsilon_\infty x}$ is the microrotation parameter,

$\beta=\frac{\eta_e}{\upsilon_\infty}$ is the reciprocal magnetic Prandtl number,

$\phi=\frac{Q(x)}{2x\rho c_p}$ is the internal heat generation or absorption parameter,

$Nr=\frac{\lambda_\infty k^*}{4\sigma^* T_3^\infty}$ is the radiation parameter

$\Pr=\frac{\upsilon_\infty\rho c_p}{\lambda_\infty}$ is the Prandtl number , $Ec=\frac{U_0^2}{c_p\left(T_w-T_\infty\right)}$ is the Eckert number,

$\mathrm{Re}=\frac{U_0 x}{\upsilon_\infty}$ is the Reynolds number.

The transformed boundary conditions are,

$$f(0)=F,\ f'(0)=1, g\left(0\right)=0,\ H(0)=H''(0)=0,\ \theta\left(0\right)=1,$$

$$f'(\infty)=0\,,g(\infty)=0,H'(\infty)=1,\ \theta(\infty)=0 \qquad (16)$$

Where $F=-\dfrac{v_w}{\sqrt{2x/(\upsilon_\infty U_0)}}$ is the dimensionless suction or injection velocity such that F>0 indicates fluid wall suction and F<0 indicates fluid wall blowing or injection.

The important physical quantities of our interest in the problem are the skin friction co-efficient (c_f) and Nusselt number (Nu) which are defined as,

$c_f=\dfrac{2\tau_w}{\rho U_0^2}$ where the shear stress at the surface is given by, $\tau_w=[(\mu+k)\dfrac{\partial u}{\partial y}+kN]_{y=0}$,

$Nu=\dfrac{xq_w}{\lambda_\infty(T_w-T_\infty)}$, where the heat flux is given by , $q_w=-\lambda[\dfrac{\partial T}{\partial y}]_{y=0}$,

Therefore,

$$c_f(2\,\mathrm{Re})^{1/2}=(\frac{2\theta_r}{\theta_r-1}+K_1)f''(0)$$

$$Nu\,\mathrm{Re}^{-1/2}\,2^{1/2}=\frac{\theta_c}{1-\theta_c}\theta'(0)$$

III. RESULT AND DISCUSSION

The non-linear ordinary differential equations (12)-(15) with the boundary conditions (16) is solved numerically by using the fourth order Runge-Kutta method along with the shooting technique. The numerical values of various parameters are taken as θ_c = -5, θ_r =-5, Nr=10, M=.1, Pr=.7, Ec=.001, K=.1, G=2, ϕ=.001, β=2, F=.5/-.5 unless otherwise stated. The numerical computations have been carried out by developing codes for MATLAB and results are presented graphically to get a physical insight of the problem for the dimensionless velocity profile f′(η), dimensionless microrotation profile g(η), temperature profile θ(η) and induced magnetic field profile H′(η) with the variation of different parameters in figures (1-18). Figures 1-4 display the effect of variable viscosity parameter θ_r on velocity, microrotation, temperature and induced magnetic field profiles respectively. It can be seen from figure 1 that an increase in θ_r lowers the velocity profile. An increase in the magnitude of θ_r corresponds to an increase in the value of (T_w - T_∞) which eventually decreases the time of interaction between the neighboring molecules and the intermolecular forces between the fluid and subsequently, causes an increase in the viscosity of the fluid which leads to the fluid moving slower for both suction and injection. From figure 2 it is seen that microrotation profile enhances with the increase of θ_r due to the elastic property of the micropolar fluid. From figure 3 it is observed that temperature profile increases with the increasing values of θ_r. As (T_w - T_∞) increases the fluid expands as it is an incompressible fluid. Thus the fluid assumes all the heat energy available and as a result temperature profiles increase. From figure 4 it is seen that induced magnetic field profiles decreases for both suction and injection significantly as θ_r increases. Figures 5-8 exhibit the effects of induced magnetic field parameter M on velocity, microrotation, temperature and induced magnetic field profiles respectively. From the figure 5, it is observed that velocity profiles reduce considerably with the increasing values of M due to the effect of Lorentz force which opposes the flow. From figure 6 it is clearly seen that microrotation profile increases with the increasing values of M because as M increases the Lorentz force increases so that temperature of the fluid enhanced and molecules get released from their bonds holding them and as a result rotation of the fluid elements increase for both suction and injection. Figure 7 shows that temperature profiles enhanced with the increasing values of M as Lorentz force increases temperature of the fluid enhanced. For both suction and injection, induced magnetic field profile reduces with the increasing values of M which is clearly observed from figure 8. Figures 9-12 indicate the effects of coupling constant parameter K on velocity, microrotation, temperature and induced magnetic field profiles respectively. Since coupling constant parameter K is the ratio of vortex viscosity to the dynamic viscosity, so as K increases velocity and induced magnetic field profiles enhance and microrotation and temperature profiles are found to be reduced for both suction and injection as shown in figures 9-12. From the figure 13 it is seen that microrotation profile decreases with the increasing values of microrotation parameter G. Figures 14 and 15 represent the effects of thermal conductivity parameter θ_r and Prandtl number Pr on temperature profile. Figure 14 shows that for both suction and injection temperature decreases and this is obvious as temperature is assumed to be the inverse linear function of thermal conductivity parameter. From figure 15 it is clearly observed that temperature profile reduces with the increasing values of Pr. As by definition Prandtl number is the ratio of the viscous force to the thermal diffusion and hence the result is obvious. Figures 16 and 17 indicate the effects of reciprocal magnetic Prandtl number β on induced magnetic field for suction and injection respectively and it is seen that for both suction and injection induced magnetic field reduces. From figure 18 it is found that microrotation increases with the increasing values of β.

Tables1-2 display the missing values of f″(0), g′(0), θ′(0) and H″(0) for various values of M, θr, K and θc respectively. It has been observed that with the increasing values of θ_r and M the values of f″(0) and H″(0) reduce but opposite trend is seen with g′(0) and θ′(0). With the increasing values of θ_c the values of f″(0), g′(0) and θ′(0) decrease while H″(0) increases. The values of f″(0) and H″(0)increase and that of g′(0) and θ′(0) decrease with the increasing values of K.

Figures 1-18 and tables 1-2 are displayed below:

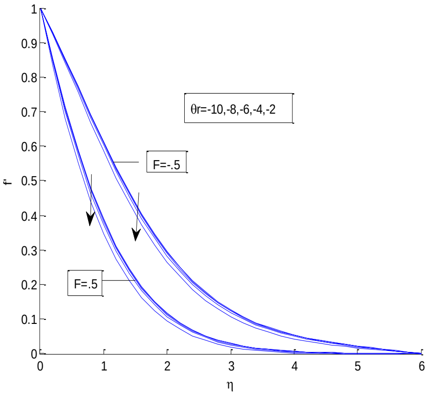

Fig 1: Velocity profile for different θ_{r}

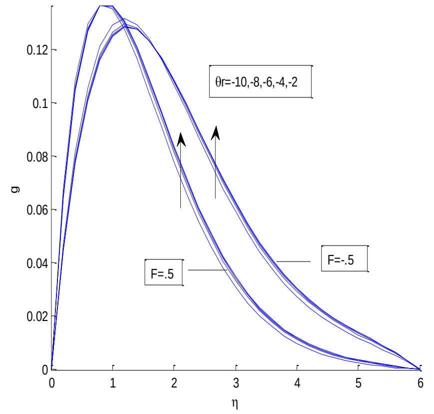

Fig 2: Microrotation profile for different θ_{r}

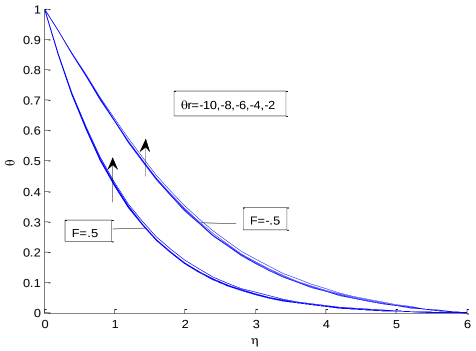

Fig 3: Temperature profile for different θ_{r}

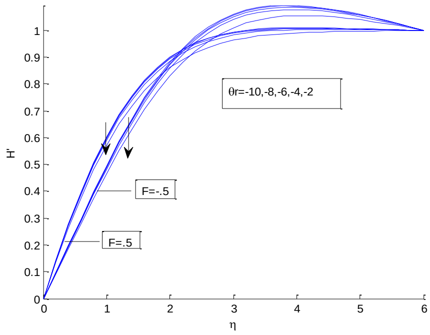

Fig 4: Induced magnetic field profile for different θ_{r}

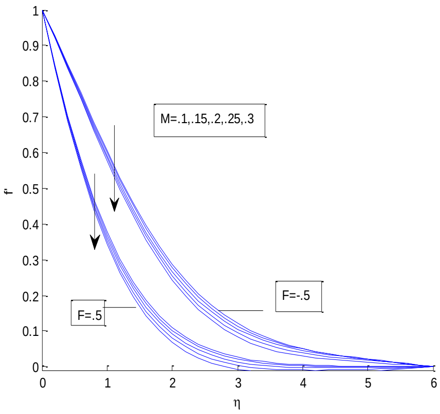

Fig 5: Velocity profile for different M

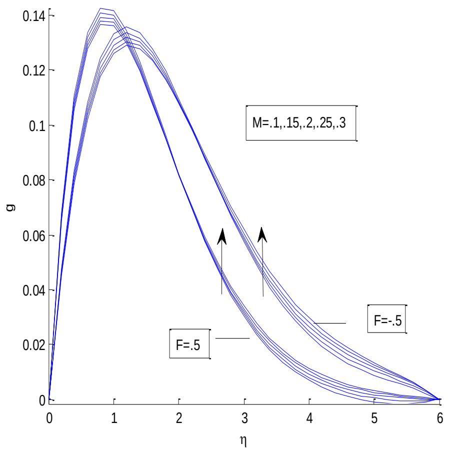

Fig 6: Microrotation profile for different M

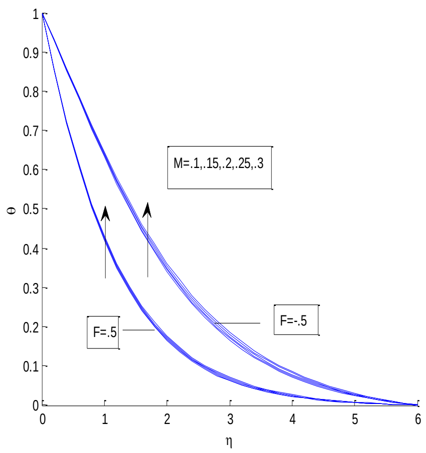

Fig 7: Temperature profile for different M

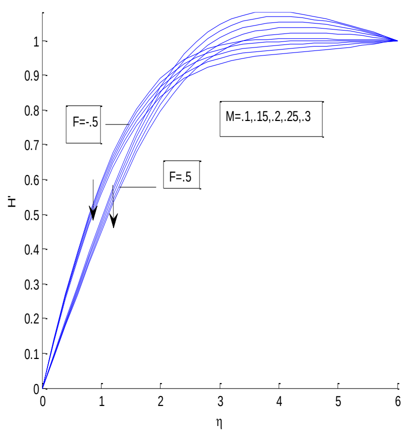

Fig 8: Induced magnetic field profile for different M

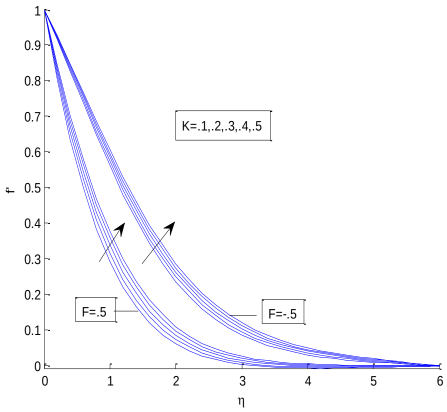

Fig 9: Velocity profile for different K

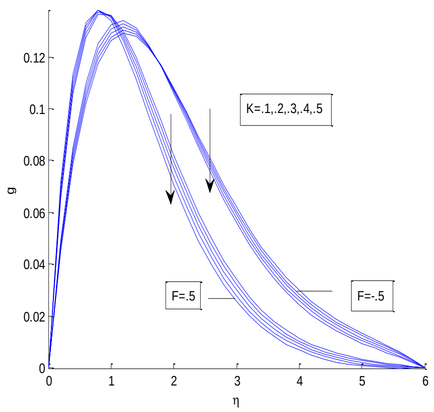

Fig 10: Microrotation profile for different K

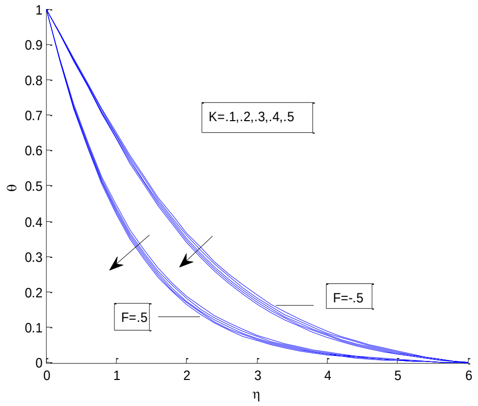

Fig 11: Temperature profile for different K

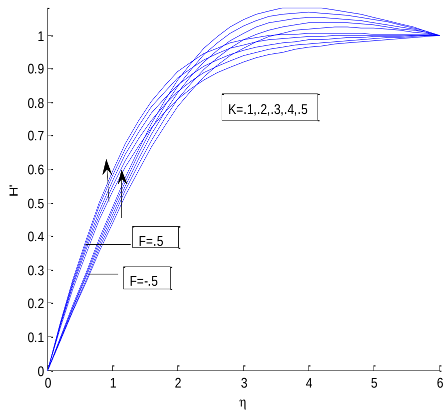

Fig 12: Induced magnetic field profile for different K

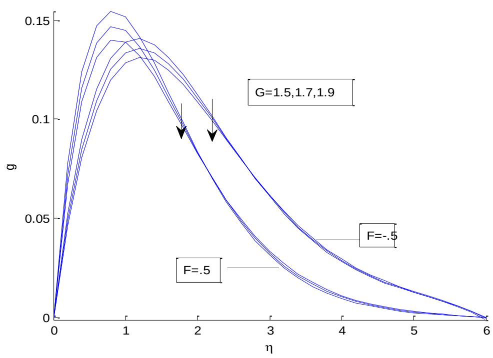

Fig 13: Microrotation profile for different G

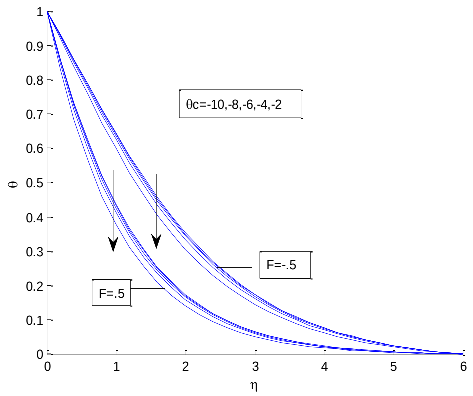

Fig 14: Temperature profile for different θ_c

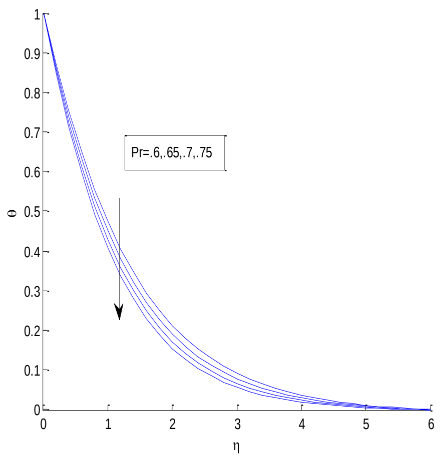

Fig 15: Temperature profile for different P_r

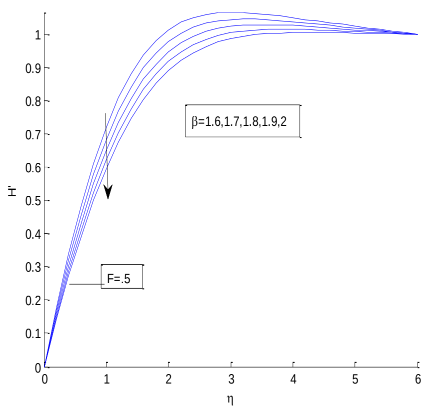

Fig16: Induced magnetic field profile for different β

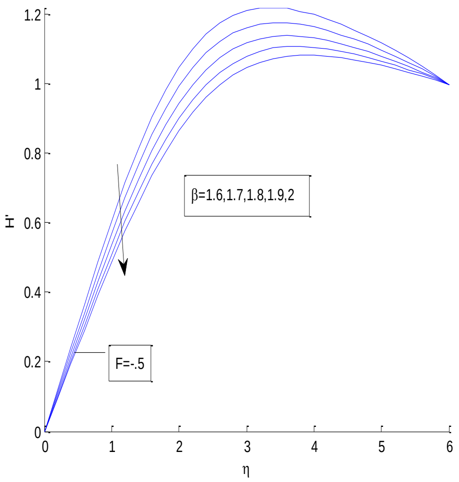

Fig 17: Induced magnetic field profile for different β

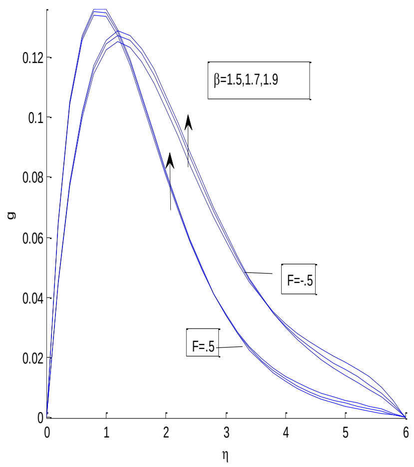

Fig 18: Microrotation profile for different β

Table1: Estimated missing values of f´´(0) , g´(0), θ´(0) and H″(0)for various θ_r and M and θ_c = -5 , Pr=.7, Ec=.001, K=.1, G=2, Nr=10, β=2,ϕ=.001, F=.5

M	θr	f″(0)	g′(0)	θ′(0)	H″(0)
.1	-10	-0.76664	0.391661	-0.77634	0.730568
	-8	-0.77508	0.393618	-0.77524	0.72825
	-6	-0.78874	0.396795	-0.77344	0.724484
	-4	-0.81463	0.40285	-0.76999	0.717304
	-2	-0.88252	0.418948	-0.76069	0.69821
.2	-10	-0.78468	0.39972	-0.77129	0.717185
	-8	-0.79344	0.4018	-0.77009	0.714708
	-6	-0.80762	0.405179	-0.76814	0.710683
	-4	-0.83453	0.411627	-0.76439	0.703003
	-2	-0.90522	0.428814	-0.75422	0.682541
.3	-10	-0.80388	0.408534	-0.76561	0.70301
	-8	-0.81298	0.410746	-0.76431	0.700372
	-6	-0.82771	0.414342	-0.76218	0.696086
	-4	-0.85569	0.42121	-0.75809	0.6879
	-2	-0.92934	0.439566	-0.74693	0.666049
.4	-10	-0.82414	0.418056	-0.75929	0.688262
	-8	-0.83359	0.420407	-0.75787	0.685465
	-6	-0.8489	0.424231	-0.75555	0.680919
	-4	-0.87798	0.431542	-0.75107	0.67223
	-2	-0.95469	0.451133	-0.73881	0.648997

Table 2: Estimated missing values of f´´(0) , g´(0), θ´(0) and H″(0)for various θ_c and K and θ_r = -5 , Pr=.7, Ec=.001, M=.1, G=2, Nr=10, β=2,ϕ=.001, F=.5

K	θc	f″(0)	g′(0)	θ′(0)	H″(0)
.1	-10	-1.04114	0.454416	-0.69306	0.656355
	-8	-1.04126	0.454345	-0.70489	0.65647
	-6	-1.04145	0.454229	-0.72436	0.656656
	-4	-1.04183	0.454007	-0.76244	0.657011
	-2	-1.04284	0.453405	-0.87038	0.657954
.2	-10	-0.96473	0.438412	-0.70219	0.674191
	-8	-0.96483	0.438347	-0.71412	0.674301
	-6	-0.96501	0.438241	-0.73375	0.674478
	-4	-0.96534	0.438038	-0.77214	0.674815
	-2	-0.96624	0.43749	-0.88088	0.675711
.3	-10	-0.90067	0.424099	-0.7103	0.690853

	-8	-0.90076	0.424039	-0.72231	0.690957
	-6	-0.90092	0.423942	-0.74209	0.691126
	-4	-0.90122	0.423755	-0.78074	0.691447
	-2	-0.90203	0.423252	-0.89017	0.692299
.4	-10	-0.8461	0.411189	-0.71756	0.706481
	-8	-0.84619	0.411134	-0.72965	0.706581
	-6	-0.84633	0.411044	-0.74954	0.706742
	-4	-0.84661	0.410872	-0.78843	0.707048
	-2	-0.84734	0.410409	-0.89847	0.707858

IV. CONCLUSION

Numerical study of the effects of variable viscosity and thermal conductivity on the boundary layer flow and heat transfer of a MHD micropolar fluid past a stretched semi-infinite vertical plate under induced magnetic field in presence of heat generation or absorption, thermal radiation and viscous dissipation effects is investigated and we arrive at the following significant observations:

1. Velocity decreases with the increasing values of viscosity parameter and magnetic parameter; but reverse trend is observed in case of coupling constant parameter for both suction and injection.
2. In case of microrotation, it is observed that microrotation increases with the increasing values of both viscosity parameter and magnetic parameter; but it decreases with the increasing values of coupling constant parameter and microrotation parameter for both suction and injection.
3. Temperature decreases with the increasing values thermal conductivity parameter, coupling constant parameter, Prandtl number and radiation parameter whereas it increases with the increasing values of viscosity parameter and magnetic parameter for both suction and injection.
4. Induced magnetic field profile decreases with viscosity parameter, magnetic parameter and coupling constant parameter ; but reverse trend is observed in case of reciprocal magnetic Prandtl number for both suction and injection.

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