Some New Results On Ladder Graphs

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Abstract: A graph G for which two vertices of a graph are said to be radial to each other if the distance between them is equal to the radius of the graph. The radial graph of a graph G, denoted by R(G), has the vertex set as in G and then two vertices are adjacent in R(G) if and only if they are radial to each other in G. In This paper we have determined the radial graph of ladder graphs. The geodetic polynomials and detour geodetic polynomials of ladder graphs are derived and some important results are proved.

Keywords: Distance, Detour geodetic polynomial, Geodetic polynomial, Ladder graph, Radial graph.

AMS Classification : 05C12, 05C60, 05C75

I. Introduction

In this paper we discuss only finite simple and connected graph. For basic graph theoretical terminology we refer [1]. In [5] the concept of radial graph R(G) is introduced and the characterization for R(G) is proved . The concept of Geodetic polynomials of a graph using Geodetic sets of a graph are introduced in [8]. Geodetic polynomial, Detour geodetic polynomial of some radial graphs are discussed in [6]. Here we have derived some results, on radial graph of ladder graphs and geodetic polynomial, detour geodetic polynomial of ladder graphs .

1.1. Preliminaries

For a graph G, the distance d(u,v) between a pair of vertices u and v is the length of a shortest path joining them. The eccentricity e(u) of a vertex u is the distance to a vertex farthest from u. The radius r(G) of G is defined as the minimum eccentricity of all the vertices of G and the diameter d(G) of G is defined as the maximum eccentricity of all the vertices of G.

A graph G for which r(G) = d(G) is called a self centred graph. Two vertices of a graph are said to be radial to each other if the distance between them is equal to the radius of the graph. The radial graph of a graph G, denoted by R(G), has the vertex set as in G and then two vertices are adjacent in R(G) if and only if they are radial to each other in G.

II. Radial graph of Ladder Graphs

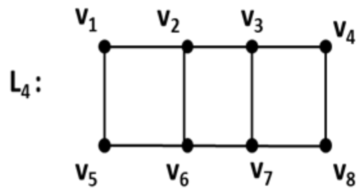
In this section we discuss radial graph of ladder graphs and proved some theorems for finding the radial graphs of ladder graphs **.**

Definition 2.1

The ladder graph L_n is a planar undirected graph with 2n vertices and 3n-2 edges.The ladder graph can be obtained as the Cartesian product of two path graphs, one which has only one edge: $L_{n,1} = P_n \times P_2$.

Example 2.2

The following is the example for ladder graph L_4 with 8 vertices.



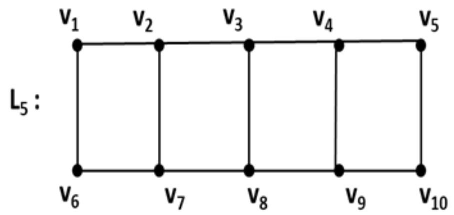
Theorem 2.3

Let L_n be the ladder graph with 2n vertices $n \geq 4$. Then the radial graph of ladder graph will have 4 vertices of degree n-2 and all other vertices are of degree n-3.

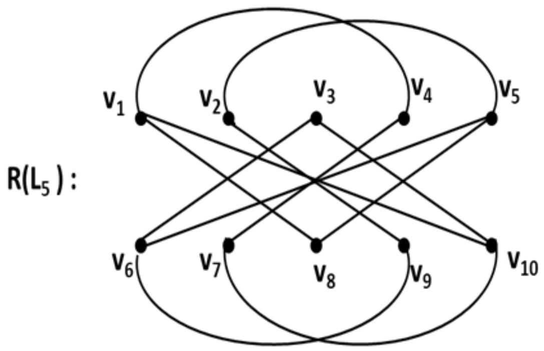
Proof:

Let us prove the theorem by induction on the number of vertices.

Case (i) If n is odd, $n \geq 4$.Let n=5 then L_5 is a ladder graph with 10 vertices and it will be of the form,

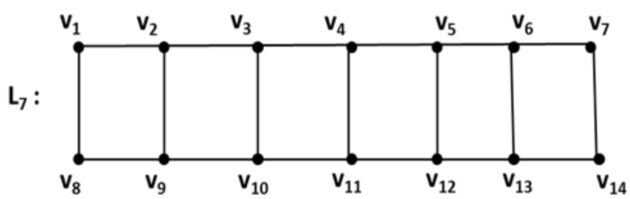


The radial graph of the ladder graph L_5 is

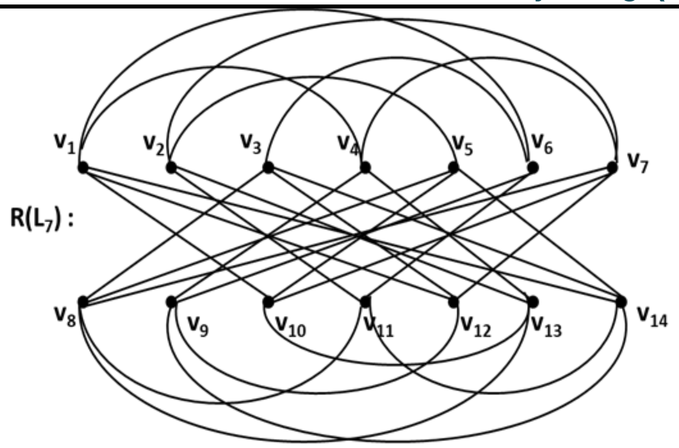


Here the vertices v_1, v_5, v_6 and v_{10} has degree 3 and other 6 vertices of degree 2.

If n = 7, L_7 is a ladder graph with 14 vertices and it is,



The radial graph of the above ladder graph is



In the above graph we observe that the vertices v_1, v_7, v_8 and v_{14} has degree 5 . All

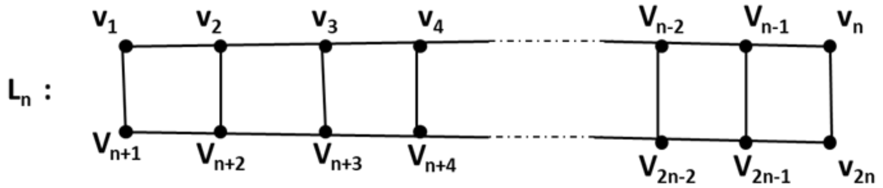
other vertices has degree 4 .

The theorem is true for n = 5 and n = 7.

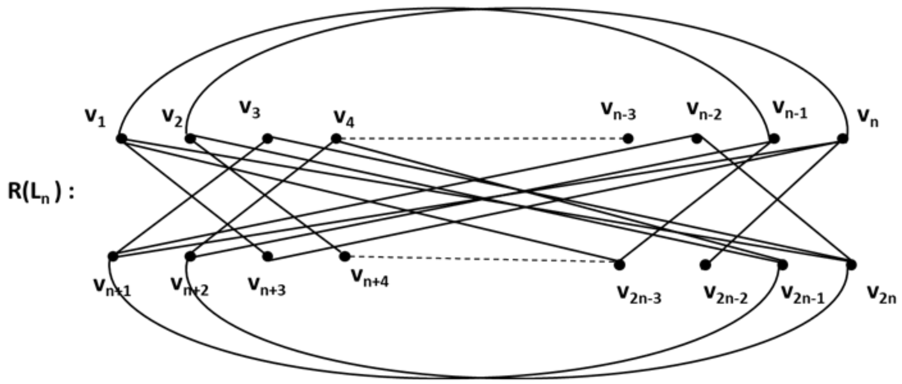
Let us assume that the theorem is true for all ladder graph with n-1 vertices and n is odd. (i.e) The radial graph of the ladder graph L_{n-1} has 4 vertices of degree n-2 and all other vertices has degree n-3.

Now we prove the theorem for ladder graph with n vertices.

Let L_n is the ladder graph with 2n vertices,n is odd.



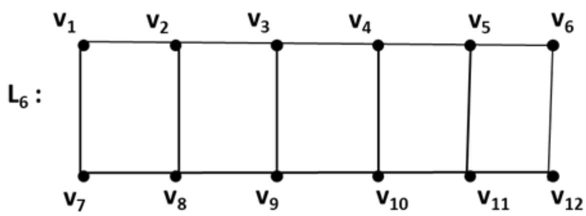
The radial graph of the ladder graph L_n is



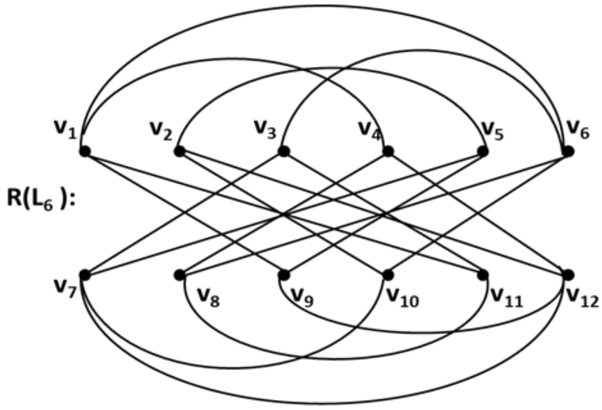
From the above radial graph of ladder graph L_n , we observe that the degree of all inner vertices is less than the degree of the corner vertices $v_1, v_n, v_{n+1}, v_{2n}$ .Hence the radial graph of the ladder graph has 4 vertices of degree n-2 and all other vertices has degree n-3.

Case (ii) Assume that n is even and n = 6

Let L_6 is the ladder graph with 12 vertices.

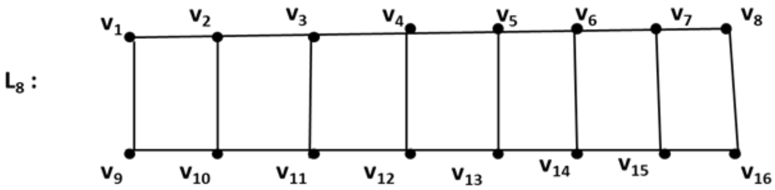


The radial graph of the ladder graph L_6 is

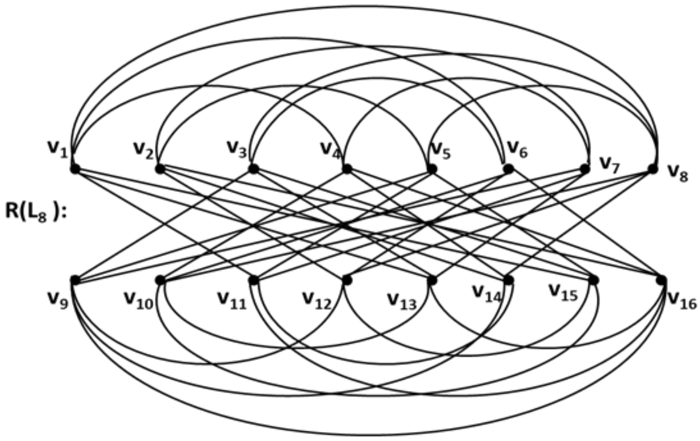


Here the vertices v_1,v_6,v_7,v_{12} has degree 4 (i.e n-2)and all other vertices has degree 3(i.e n-3).

If n = 8, Let L_8 is the ladder graph with 16 vertices and is



The radial graph of the above ladder graph is



Here the vertices v_1,v_8,v_9,v_{16} has degree 6 and all other vertices has degree 5.

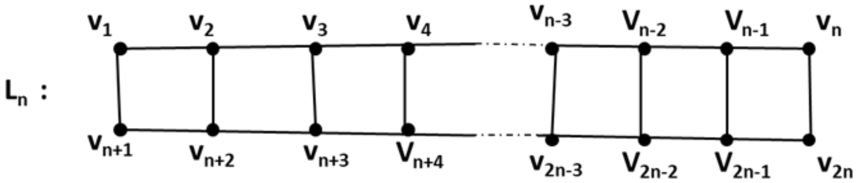
The theorem is true for n = 6 and n = 8.

Let us assume the theorem is true for all the ladder with n-1 vertices and n is even (i. e) the radial graph of ladder graph

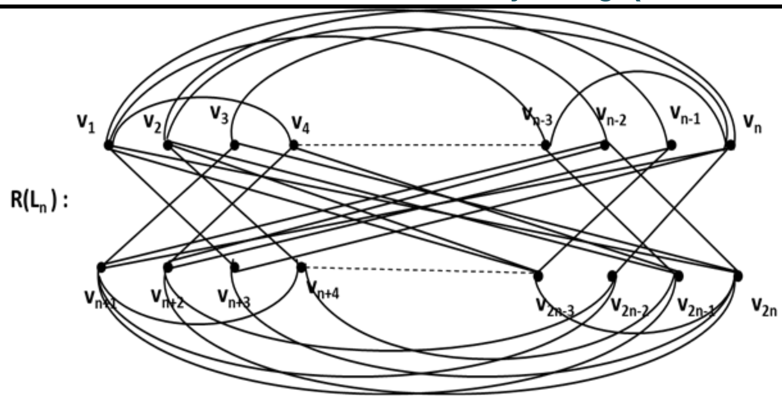
L_{n-1} has 4 vertices of degree n-2 and all other vertices has degree n-3.

Now we prove the theorem for ladder graph with n vertices.

Let L_n is the ladder graph with 2n vertices and n is even.



The radial graph of the above ladder graph is



From the above radial graph of ladder graph L_n , we observe that the degree all inner vertices is less than the degree of the corner vertices v_1,v_n,v_{n+1},v_{2n} .Hence the radial of the ladder graph has 4 vertices of degree n-2 and all other vertices has degree n-3.

III. Geodetic Polynomial of Ladder Graphs

In this section we discuss geodetic polynomial of ladder graphs

Definiton 3.1

Let $\mathcal{G}$ (G,i) be the family of geodetic sets of the graph G with cardinality i and let

$g_e\,(G,i) = |\,\mathcal{G}\,(G,i)|$. Then the geodetic polynomial $\mathcal{G}\,(G,x)$ of G is defined as $\mathcal{G}\,(G,x) = \sum_{i=\,g(G)}^{|V(G)|} g_e\,(G,i)\,x^i$ where $g(G)$ is the geodetic number of G.

Theorem 3.2

The geodetic polynomial of L_n, if n $\geq$ 2 is $\mathcal{G}$ (L_n ,x) = $(x+1)^{2n} - (2nx+1)$.

Proof:

Let L_n be a ladder graph with 2n vertices without loss of generality we choose n ≥ 2.

Let X={ $u_1, u_2, u_3, \ldots\ldots\ldots u_n$ }. The only geodetic set with minimum cardinality is 2 in X. Therefore $g_e\,(L_n,2) = 2\,nC_2$.The geodetic set with cardinality 3 are $g_e\,(L_n,3) = 2nC_3$.

$\mathcal{G}\,(L_n,x) = 2nC_2\,x^2 + 2nC_3\,x^3 + \ldots\ldots\ldots\ldots + 2nC_{2n}\,x^{2n}$

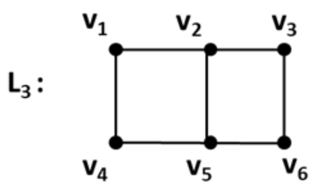
$\mathcal{G}\,(L_n,x) = 2nC_2\,x^2 + 2nC_3\,x^3 + \ldots\ldots\ldots\ldots + 2nC_{2n}\,x^{2n} + 2nx + 1 - 2nx - 1$

$= 2nC_2\,x^2 + 2nC_3\,x^3 + \ldots\ldots\ldots\ldots + 2nC_{2n}\,x^{2n} + 2nx + 1 - (2nx+1)$.

$\mathcal{G}\,(L_n,x) = (1+x)^{2n} - (2nx+1)$.

Example 3.3

Let L_3 is the ladder graph with 6 vertices**.**



$g_e\,(L_3,2) = \{$ (v_1 v_2), (v_1 v_3), (v_1 v_4), (v_1 v_5), (v_1 v_6), (v_2 v_3), (v_2 v_4), (v_2 v_5), (v_2 v_6), (v_3 v_4), (v_3 v_5), (v_3 v_6), (v_4 v_5), (v_4 v_6), (v_5 v_6)}

$|g_e\,(L_3,2)|$ = 15

$g_e\ (L_3, 3) =$ { ($v_1\ v_2\ v_6$), ($v_1\ v_3\ v_6$), ($v_1\ v_4\ v_6$), ($v_1\ v_5\ v_6$), ($v_1\ v_2\ v_5$), ($v_1\ v_3\ v_5$), ($v_1\ v_4\ v_5$), ($v_1\ v_2\ v_4$), ($v_1\ v_3\ v_4$), ($v_1\ v_2\ v_3$), ($v_2\ v_3\ v_4$), ($v_2\ v_3\ v_5$), ($v_2\ v_4\ v_5$), ($v_2\ v_3\ v_6$), ($v_2\ v_4\ v_6$), ($v_2\ v_5\ v_6$), ($v_3\ v_4\ v_6$), ($v_3\ v_5\ v_6$), ($v_3\ v_4\ v_5$), ($v_4\ v_5\ v_6$)}

$|g_e\ (L_3, 3)| = 20$

$g_e\ (L_3, 4) =$ { ($v_1\ v_2\ v_3\ v_6$), ($v_1\ v_2\ v_4\ v_6$), ($v_1\ v_2\ v_5\ v_6$), ($v_1\ v_3\ v_4\ v_6$), ($v_1\ v_3\ v_5\ v_6$), ($v_1\ v_4\ v_5\ v_6$), ($v_2\ v_3\ v_4\ v_6$), ($v_2\ v_3\ v_5\ v_6$), ($v_2\ v_4\ v_5\ v_6$), ($v_3\ v_4\ v_5\ v_6$), ($v_1\ v_2\ v_3\ v_5$), ($v_1\ v_2\ v_4\ v_5$), ($v_1\ v_3\ v_4\ v_5$), ($v_2\ v_3\ v_4\ v_5$), ($v_1\ v_2\ v_3\ v_4$) }

$|g_e\ (L_3, 4)| = 15$

$g_e\ (L_3, 5) =$ {($v_1\ v_2\ v_3\ v_4\ v_6$), ($v_1\ v_2\ v_3\ v_4\ v_5$),($v_1\ v_2\ v_3\ v_5\ v_6$), ($v_1\ v_2\ v_4\ v_5\ v_6$), ($v_1\ v_3\ v_4\ v_5\ v_6$), ($v_2\ v_3\ v_4\ v_5\ v_6$) }

$|g_e\ (L_3, 5)| = 6$

$g_e\ (L_3, 6) =$ { ($v_1\ v_2\ v_3\ v_4\ v_5\ v_6$) }

$|g_e\ (L_3, 6)| = 1$

$\mathcal{G}\ (L_3, x) = \sum_{i=g(G)}^{|V(G)|} g_e\ (L_3, i)\ x^i$

$\mathcal{G}\ (L_3, x) = g_e\ (L_3, 2)\ x^2 + g_e\ (L_3, 3)\ x^3 + g_e\ (L_3, 4)\ x^4 + g_e\ (L_3, 5)\ x^5 + g_e\ (L_3, 6)\ x^6$

$\mathcal{G}\ (L_3, x) = 15\ x^2 + 20x^3 + 15x^4 + 6\ x^5 + x^5$.

The geodetic polynomial of L_3 is $\mathcal{G}\ (L_3, x) = 15\ x^2 + 20x^3 + 15x^4 + 6\ x^5 + x^5$.

IV. Detour geodetic polynomial of Ladder Graphs

In this section we find detour geodetic polynomial of ladder graphs

Defintion 4.1:

Let D $\mathcal{G}$ (G,i) be the family of detour Geodetic sets of the graph G with cardinality i and let

$Dg_e\ (G, i) = |D\mathcal{G}\ (G, i)|$. Then the Detour geodetic polynomial $D\mathcal{G}\ (G, x)$ of G is defined as

$D\mathcal{G}\ (G, x) = \sum_{i=dg(G)}^{dg+(G)} Dg_e\ (G, i)\ x^i$ Where $d_g(G)$ is the Detour number of G.

Theorem 4.2

The Detour geodetic polynomial of the ladder graph L_n is $n(2n-1)x^2$

i.e D $\mathcal{G}$ (L_n,x) = $n(2n-1)x^2$, $n \geq 4$

Proof:

In the ladder graph L_n has 2n vertices, $n \geq 4$. $d_g(G) = 2$, $dg^+(G) = 2$ **.**

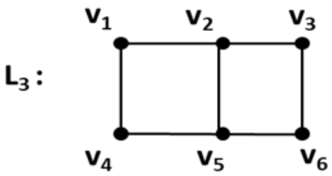
Hence, There $2nC_2$ detour geodetic set with cardinality 2 . Hence the detour geodetic polynomial of the ladder graph is $n(2n-1)x^2$,$n \geq 4$.

i.e D $\mathcal{G}$ (L_n,x) = $n(2n-1)x^2$, $n \geq 4$

Hence the proof.

Example 4.3

Let L_3 is the ladder graph with 6 vertices.



$DS_1 = \{ v_1 v_2\}$ $DS_2 = \{ v_1 v_3\}$ $DS_3 = \{ v_1 v_4\}$, $DS_4 = \{ v_1 v_5\}$, $DS_5 = \{ v_1 v_6\}$, $DS_6 = \{ v_2 v_3\}$, $DS_7 = \{ v_2 v_4\}$, $DS_8 = \{ v_2 v_5\}$, $DS_9 = \{ v_2 v_6\}$, $DS_{10} = \{ v_3 v_4\}$, $DS_{11} = \{ v_3 v_5\}$, $DS_{12} = \{ v_3 v_6\}$, $DS_{13} = \{ v_4 v_5\}$, $DS_{14} = \{ v_4 v_6\}$, $DS_{15} = \{ v_5 v_6\}$.

$Dg_e\ (L_4, 2) = \{ \{ v_1 v_2\},\{ v_1 v_3\} , \{ v_1 v_4\},\{ v_1 v_5\},\{ v_1 v_6\},\{ v_2 v_3\},\{ v_2 v_4\}, \{ v_2 v_5\}, \{ v_2 v_6\}, \{ v_3 v_4\}, \{ v_3 v_5\},\{ v_3 v_6\}, \{ v_4 v_5\}, \{ v_4 v_6\}, \{ v_5 v_6\} \}$

$|Dg_e\ (L_4, 2)| = 15$

$d_g(L_4) = 2$,$dg^+(L_4) = 2$

Detour geodetic polynomial of ladder Graph L_4 is

$D\,\mathcal{G}\,(L_4, x) = \sum_{i=2}^{2} Dg_e\ (L_4, i)\ x^i$

$D\,\mathcal{G}\,(L_4, x) = 15x^2$

The detour geodetic polynomial of ladder graph L_4 is

$D\,\mathcal{G}\,(L_4, x) = 15x^2$.

Conclusion

Here Radial graph of ladder graph and geodetic polynomial, detour geodetic polynomial of ladder graphs have been studied. Further we can find the geodetic polynomial of other graphs.

REFERENCES:

[1] Buckley. F, Harary. F, Distance in graphs, Addition Wesley, Read city,CA,1990.

[2] Chartrand.G,and Zhang.PDistance in graphs,In Hand Book of graph theory(ed.by.J.Gross and J.Yellen),CRC Press,Bocaraton,FL(2004), PP 873-788

[3] Chartrand. C, and P. Zang,Distance in graphs-Taking a longview,AKCE. J.Graphs,Cokbin,1,1(2004), PP 1-13.

[4] Chartrand. G ,H. Escuadro and P .Zang Detour distance in graphs,J.Combin.Math.Comput.53(2005), PP 75-94.

[5] Kathiresan .K.M and Marimuthu. G, A Study on radial graphs, ARS COMBINATORIA 96(2010), PP 353-356.

[6] Ramesh.G and Sathiya .A, Geodetic polynomial and Detour geodetic polynomial of Radial graphs,International journal for research in applied science and Engineering Technology, Volume 6 Issue II, Feb (2018), PP 1661-1668.

[7] Santhakumaran A.P, Detour number of a graph,NCGTGA-2012, PP 25-37.

[8] Vijayan. A and Binuselin. T, An Introduction to Geodetic Polynomial of a graph,Bulletin of Pure and Applied Science volume 31 e(Math & Stat).Issue(no.1) 2012, PP 25-32.

[9] Chartrand. C, Harary. F Geodetic sets in graphs Discussiones Mathematicae Graph Theory 20 (2000), PP 129-138.