

SELECTION OF OPTIMUM ALUMINIUM HYBRID METAL MATRIX COMPOSITE USING FUZZY AHP-VIKOR METHOD

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Abstract: The aim of this paper is to evaluate the optimum aluminium hybrid metal matrix composite in terms of mechanical and physical characterizations using Fuzzy AHP-VIKOR method. In order to evaluate the optimal composite, the aluminium hybrid metal matrix composites were fabricated according to L_{18} Taguchi orthogonal array, designed by considering the three levels of matrix materials, two levels of hybrid reinforcements and three levels of weight percent of reinforcements. The fabrication of aluminium hybrid metal matrix composites (AHMMCs) have been done by using stir casting process and to evaluate the optimal AHMMC, the tensile strength, hardness, density and porosity were considered as mechanical and physical characterization of hybrid composites. Finally the fuzzy AHP-VIKOR results inferred the 9% of silicon carbide along with flyash reinforced AA5083 composite was an optimal AHMMC material in terms of its mechanical and physical characterization. Thereafter, the microstructural studies were also been performed on optimal AHMMC using scanning electron microscope (SEM) to examine the uniformity of particle distribution in matrix and SEM results shown that uniform dispersed of reinforcement particles with no void.

IndexTerms - Hybrid optimization method, hybrid metal matrix composites, mechanical and physical characterizations, stir casting process and scanning electron microscope.

I. INTRODUCTION

In current engineering field, application of new materials has been increased and that material requires the properties like stronger, harder, light-in-weight and less expensive characteristics. Therefore, in order to attain the current engineering application, the research on hybrid metal matrix composite has become a significant area in metallurgical sciences. AHMMC is an aluminium hybrid metal matrix composite material consisting of two or more dissimilar reinforcements, in form or material composition which is amalgamated into aluminium matrix and that beacon the hybrid metal matrix composites. These two dissimilar reinforcements are essentially do not melt each other and to maintain their absolute identity for attaining the combined properties of matrix (ductility, stiffness and toughness) and reinforcements (high strength, high temperature resistance, low density, wear resistance and high modulus). The aluminium hybrid metal matrix composites can often applied in various engineering fields such as military industries, aero and space research industries, automotive sector and marine industries, etc due to their superior metallurgical characteristics.

Nowadays, the usage of industrial waste as a secondary reinforcement in the production of hybrid composites is gaining more significance. The major advantages of using this waste as one of the reinforcement, is reducing the cost of composites, readily available and lower the densities of composites. The industrial waste ie, flyash was an environment pollutant and usage of flyash along with alumina and silicon carbide will improved the mechanical properties and reduce the density of hybrid metal matrix composites [1, 17]. Among several methods available to produce the metal matrix composites, liquid metallurgy route ie, stir casting process is the cheapest production method [23]. During the fabrication of MMC, need to have awareness about the various technical challenges offered by stir casting method and S. Balasivanandha Prabu et al (2006) explored those technical problems in production of MMC with use of stir casting process. In order to eradicate those technical problems offered in casting of MMC, [19, 10, 3, 28] reported the methods to fabricate the optimum metal matrix composites. In the process of material performance improvisation, selection of reinforcing materials play a critical role and [6, 9, 11 and 4] stated that synthetic ceramic particles such as SiC and Al_2O_3 are the most influential reinforcing materials on mechanical properties of metal matrix composites (MMCs).

On the other hand selection of AHMMCs has become a very crucial role in performance improvement for many engineering application. Therefore, many researchers have been reported many scientific approaches to select an optimum materials or machining process to improve the performance of engineering applications etc. For paradigm, [2, 7] used traditional analytic hierarchy process (AHP) for selection of most appropriate matrix material to produce improved property aluminium hybrid metal matrix composites. However, due to the vagueness and uncertainty existing in AHP to weighting the importance, AHP seems to be insufficient and imprecise. Therefore [18, 8] used a Fuzzy AHP to determination of weights to the customer requirements in quality function deployment and the supplier selection for performance improvement of an organization. Many researchers was also used extent analysis method on fuzzy AHP for weighting or selecting the criterion and alternatives, but [30] showed the misapplication of the extent analysis method to fuzzy AHP problems with examples. Other than fuzzy AHP, VIKOR method under fuzzy environment was also used to solve the multi criteria decision making problems in selection of material and weighting the criteria and etc by [15, 29] and [13] reported the major advantages of VIKOR approach in material selection.

In accordance with the best of my knowledge, no work has been taken place to optimize the aluminium hybrid metal matrix composites in terms of mechanical and physical properties using FAHP-VIKOR method so far. Therefore the aim of the present study is to contribute to the decision making process in evaluation of optimum aluminium hybrid metal matrix composites (AHMMCs) using fuzzy AHP-VIKOR approach. The aim is also to work towards the prevention of uncertainty in decision making of MCDM problems by using the fuzzy analytical hierarchy process (FAHP) integrated with the VIKOR method.

II. EXPERIMENTATION

2.1 Materials

In the present study wrought aluminium alloys, 53 μ m size of SiC and Al₂O₃ and 53-106 μ m avg. size of class F hallow sphere flyash particles were used as the matrix and reinforcement constituents to produce the aluminium hybrid metal matrix composites according to Taguchi orthogonal array. Table 1 shows the material factors and their levels used to design the Taguchi orthogonal array for fabricate the aluminium hybrid metal matrix composites with optimal conditions.

Table 1 Material Factors and their levels

S. No.	Material Factors	Levels		
		1	2	3
1	Matrix Materials (MM)	AA5083	AA6082	AA7075
2	Reinforcement Materials (RM)	SiC+FA	Al ₂ O ₃ +FA	---
3	Percent of Reinforcement Materials (PRM)	3%	6%	9%

In order to reduce the experimentation cost, Taguchi orthogonal array L₁₈ experiments was obtained through the design by MiniTab software for conducting the casting experimentation. Table 2 shows the experimental design need to produce the AHMMCs with minimum cost using stir casting process.

2.2 Fabrication of AHMMC samples

Initially, the calculated amount of wrought aluminium alloys in the form of ingots was charged into the graphite crucible placed in a stir casting furnace and allowed to complete melting. After melting, the molten metal was agitated with the help of mechanical stirrer made of stainless steel, coated with zirconium material, at a speed of 600 rpm for 10 minutes to create a fine vortex for homogeneous mixture of the molten slurry. During stirring process, the preheated reinforcement particles (synthetic ceramics along with flyash) along with 2% magnesium were gradually added into the vortex of the molten metal after an effective degasify with hexachloroethane tablet. After thorough stirring, the temperature of the composite molten slurry was raised just above to equivalent melting temperature of an aluminium alloys to improve fluidity and was poured into preheated steel mould and then allowed to cool, to obtain cast composites.

Table 2 Taguchi orthogonal array design

Alternatives	MM	PRM	RM	AHMMCs
1	AA5083	3%	1.5% SiC+1.5% FA	5083/3% SiCp+FA
2	AA5083	6%	3% SiC+3% FA	5083/6% SiCp+FA
3	AA5083	9%	4.5% SiC+4.5% FA	5083/9% SiCp+FA
4	AA6082	3%	1.5% SiC+1.5% FA	5083/3% Al ₂ O ₃ +FA
5	AA6082	6%	3% SiC+3% FA	5083/6% Al ₂ O ₃ +FA
6	AA6082	9%	4.5% SiC+4.5% FA	5083/9% Al ₂ O ₃ +FA
7	AA7075	3%	1.5% SiC+1.5% FA	6082/3% SiCp+FA
8	AA7075	6%	3% SiC+3% FA	6082/6% SiCp+FA
9	AA7075	9%	4.5% SiC+4.5% FA	6082/9% SiCp+FA
10	AA5083	3%	1.5% Al ₂ O ₃ +1.5% FA	6082/3% Al ₂ O ₃ +FA
11	AA5083	6%	3% Al ₂ O ₃ +3% FA	6082/6% Al ₂ O ₃ +FA
12	AA5083	9%	4.5% Al ₂ O ₃ +4.5% FA	6082/9% Al ₂ O ₃ +FA
13	AA6082	3%	1.5% Al ₂ O ₃ +1.5% FA	7075/3% SiCp+FA
14	AA6082	6%	3% Al ₂ O ₃ +3% FA	7075/6% SiCp+FA
15	AA6082	9%	4.5% Al ₂ O ₃ +4.5% FA	7075/9% SiCp+FA
16	AA7075	3%	1.5% Al ₂ O ₃ +1.5% FA	7075/3% Al ₂ O ₃ +FA
17	AA7075	6%	3% Al ₂ O ₃ +3% FA	7075/6% Al ₂ O ₃ +FA
18	AA7075	9%	4.5% Al ₂ O ₃ +4.5% FA	7075/9% Al ₂ O ₃ +FA

The obtained cast samples were tested for property characterizations such as Tensile Strength, Hardness, Density and porosity and data are summarized in Table 3. All the experiments have been done according to ASTM standards and each value represented is an average of three measurements, except Tensile Strength and Density readings. The experiment particulars for determining the mechanical and physical properties of AHMMCs are presented in the following sections.

2.2.1 Properties characterization details of AHMMC samples

Among the various mechanical and physical properties of materials and composite materials, tensile strength, hardness, density and porosity properties are the most often considerable and evaluated characterizations. The computerized universal tensile testing machine was used to conduct the tensile test and to predict the material behaviour under tension loading on each fabricated AHMMCs as per ASTM E8 standard under room temperature. Similarly the hardness is an important property that evaluates the resistance of a material to plastic deformation, usually by penetration. The hardness test has been conducted on each fabricated specimen as per ASTM E10 standard by using Brinell hardness tester with 10 mm diameter of steel ball indenter at a load of 500 kg for a dwell period of 30 seconds. The Density of AHMMCs was determined experimentally and theoretically using Archimedes principle and rule of mixtures Eq.1.

$$\text{Density of composite} = \rho_r v_r + \rho_m v_m \quad (1)$$

Where v_m and v_r is the volume fraction of the matrix and reinforcement materials, ρ_m and ρ_r is the density of matrix reinforcement materials.

The obtained theoretical and experimental density values of AHMMCs were used to calculate the porosity levels in AHMMCs. This was obtained by using below shown equation:

$$\text{Porosity } (\epsilon) = (\rho_{td} - \rho_{md}) / \rho_{td} \quad (2)$$

Where ρ_{md} is the bulk density, ρ_{td} is the theoretical density respectively.

The maximum permissible range of porosity percent in cast metal matrix composites is documented by [16, 12] and that level has to be within 4% could be acceptable.

Table 3 Experimental results of AHMMCs

Exp. No.	Hybrid MMCs	ρ_{bd} (gm/cc)	σ_{ts} (N/mm ²)	HB (kg/mm ²)	ρ_{td} (gm/cc)	ϵ (%)
1	5083/3% SiCp+FA	2.63	353.86	134.19	2.6471	0.64
2	5083/6% SiCp+FA	2.62	362.84	176.56	2.6342	0.53
3	5083/9% SiCp+FA	2.611	371.92	220.10	2.6213	0.43
4	6082/3% SiCp+FA	2.674	181.57	101.70	2.6859	0.59
5	6082/6% SiCp+FA	2.66	193.40	144.61	2.671	0.411
6	6082/9% SiCp+FA	2.64	205.51	188.72	2.6577	0.665
7	7075/3% SiCp+FA	2.78	230.78	190.31	2.7926	0.451
8	7075/6% SiCp+FA	2.76	241.81	231.75	2.775	0.54
9	7075/9% SiCp+FA	2.74	253.05	274.31	2.757	0.616
10	5083/3% Al ₂ O ₃ +FA	2.64	345.25	109.45	2.65595	0.60
11	5083/6% Al ₂ O ₃ +FA	2.63	345.36	126.35	2.6519	0.82
12	5083/9% Al ₂ O ₃ +FA	2.625	345.39	143.69	2.64785	0.86
13	6082/3% Al ₂ O ₃ +FA	2.683	172.96	76.96	2.694	0.519
14	6082/6% Al ₂ O ₃ +FA	2.67	175.93	94.39	2.6895	0.72
15	6082/9% Al ₂ O ₃ +FA	2.673	178.92	112.31	2.6842	0.52
16	7075/3% Al ₂ O ₃ +FA	2.791	222.185	165.566	2.801	0.357
17	7075/6% Al ₂ O ₃ +FA	2.78	224.344	181.53	2.792	0.429
18	7075/9% Al ₂ O ₃ +FA	2.76	226.46	207.29	2.784	0.862

III. FUZZY ANALYTIC HIERARCHY PROCESS (FAHP)

The classical or traditional AHP method was proposed by Saaty in 1970s as a decision aid tool to setting the weights and to reduce the mental process for selection of best alternative or criteria in MCDM problems [23]. However, the application of classical AHP is insufficient for dealing MCDM problems because it uses nearly crisp decision applications, incomplete information, impreciseness of human judgments and fuzzy environment [14]. Hence, the fuzzy AHP technique was developed from classical AHP and can be called as an advanced analytical method, first proposed by P. J. M. Van Laarhoven and Pedrycz in 1983 [27], for dealing with fuzziness and uncertainty in both quantitative and qualitative criteria of MCDM problems [25]. In this method triangular fuzzy numbers (Figure 1), first explained and described as membership functions by L. A. Zadeh (1971) [31], father of fuzzy set theory, are used as a preference scale (Table 4) in spite of Saaty's scale for wipe out the fuzziness and vagueness to make a decision with precise judgments.

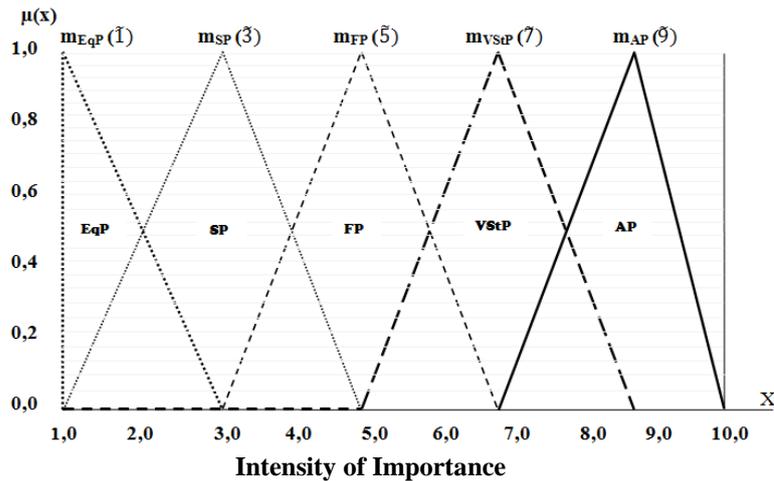


Figure 1 Comparative linguistic scale of the weights with the member functions of triangular fuzzy numbers

Table 4 Comparative linguistic scale to evaluate the weights of criteria and alternatives

Intensity of importance	Fuzzy number	Linguistic Terms	Fuzzy triangular number
1	$\tilde{1}$	Equally Preferable (EqP)	(1, 1, 2)
3	$\tilde{3}$	Slightly Preferable (SP)	(2, 3, 4)
5	$\tilde{5}$	Fairly Preferable (FP)	(4, 5, 6)
7	$\tilde{7}$	Very strongly Preferable (VStP)	(6, 7, 8)
9	$\tilde{9}$	Absolutely Preferable (AP)	(8, 9, 10)

A major contribution of fuzzy set theory is its ability of representing vague data and it also allows mathematical operators and programming to pertain to the fuzzy domain. A fuzzy set is a class of objects with a range of grades of membership. Such a set is characterized by a membership function, which assigns to each object a grade of membership ranging between zero and one. A tilde “~” will be positioned above a symbol if the symbol represents a fuzzy set. The fuzzy AHP triangular fuzzy numbers are used by [05] to explain the progress of scaling scheme in the judgment matrices and also used interval arithmetic to solve the fuzzy eigenvector.

The theoretical procedure of the FAHP method consists of four steps, are as follows:

Step 1: Comparing the performance score with triangular fuzzy numbers ($\tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}$) to indicate the relative strength of each pair of elements in the same hierarchy.

Step 2: Constructing the fuzzy comparison matrix by using triangular fuzzy numbers, via pair wise comparison, the fuzzy judgment matrix $\tilde{A}(a_{ij})$ is constructed as given below;

$$\tilde{A} = \begin{pmatrix} 1 & \tilde{a}_{12} & \dots & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \dots & \dots & \tilde{a}_{2n} \\ \tilde{a}_{31} & \dots & \dots & \dots & \tilde{a}_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \dots & \dots & 1 \end{pmatrix}$$

Where, $\tilde{a}_{ij} = 1$, if $i = j$, and $\tilde{a}_{ij} = \tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}$ or $\tilde{1}^{-1}, \tilde{3}^{-1}, \tilde{5}^{-1}, \tilde{7}^{-1}, \tilde{9}^{-1}$, if $i \neq j$.

Step 3: Solving fuzzy eigen value: A fuzzy eigen value, $\tilde{\lambda}$ is a fuzzy number solution to

$$\tilde{A}\tilde{x} = \tilde{\lambda}\tilde{x} \tag{3}$$

Where is $n \times n$ fuzzy matrix containing fuzzy numbers \tilde{a}_{ij} and \tilde{x} is a non-zero $n \times 1$, fuzzy vector containing fuzzy number \tilde{x}_i . To perform fuzzy multiplications and additions by using the interval arithmetic and α – cut, the equation $\tilde{A}\tilde{x} = \tilde{\lambda}\tilde{x}$ is equivalent to $[a_{i1}^\alpha x_{11}^\alpha, a_{i1u}^\alpha x_{1u}^\alpha] \oplus \dots \oplus [a_{inl}^\alpha x_{nl}^\alpha, a_{inu}^\alpha x_{nu}^\alpha] = [\lambda x_{il}^\alpha, \lambda x_{iu}^\alpha]$

Where,

$$\tilde{A} = [\tilde{a}_{ij}], \tilde{x}^t = (\tilde{x}_1, \dots, \tilde{x}_n)$$

$$\tilde{a}_{ij}^\alpha = [a_{ijl}^\alpha, a_{iju}^\alpha], \tilde{x}_i^\alpha = [x_{il}^\alpha, x_{iu}^\alpha], \tilde{\lambda}^\alpha = [\lambda_l^\alpha, \lambda_u^\alpha] \tag{4}$$

For $0 < \alpha \leq 1$ and all i, j , where $i = 1, 2, \dots, n, j = 1, 2, \dots, n$

α – cut is known to incorporate the experts or decision maker(s) confidence over his/her preference or the judgments. Degree of satisfaction for the judgment matrix \tilde{A} is estimated by the index of optimism μ . The larger value of index μ indicates the higher degree of optimism. The index of optimism is a linear convex combination (Yong-Han Lee et al 2003) defined as

$$\widetilde{a}_{iju}^\alpha = \mu a_{iju}^\alpha + (1 - \mu) a_{ijl}^\alpha, \forall \mu \in [0, 1] \tag{5}$$

While the α is fixed, the following matrix can be obtained after setting the index of optimism, μ , in order to estimate the degree of satisfaction.

The eigen vector is calculated by fixing the μ value and identifying the maximal eigen value. α – cut will yield an interval set of values from a fuzzy number. For example, $\alpha = 0.5$ will yield a set $\alpha_{0.5} = (2, 3, 4)$. The operation is presented by using Table 4 and Figure 1.

$$\widetilde{A} = \begin{pmatrix} 1 & \widetilde{a}_{12}^\alpha & \cdots & \cdots & \widetilde{a}_{1n}^\alpha \\ \widetilde{a}_{21}^\alpha & 1 & \cdots & \cdots & \widetilde{a}_{2n}^\alpha \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \widetilde{a}_{n1}^\alpha & \widetilde{a}_{n2}^\alpha & \cdots & \cdots & 1 \end{pmatrix}$$

Step 4: Normalization of the formulated pair wise comparisons matrix and calculation of priority weights (approx. attribute weights), and the matrices and priority weights for alternatives have to be done before calculating λ_{max} . In order to check the adequacy of FAHP method results, the consistency ratio for formulated matrices and overall consistency index are need to be done by the following equation CI, and the measure of consistency index is called the CI,

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{6}$$

The consistency ratio (CR) is used to estimate directly the consistency of pair wise comparisons. The CR is computed by dividing the CI by a value obtained from a Random Index (RI) table in Table 5;

Table 5 Random index table

N	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

$$CR = \frac{CI}{RI} \tag{7}$$

If the CR is less than 0.10, then the formulated decision matrix is acceptable, otherwise not.

In order to achieve more precise prioritization for improvise the judgment in optimization process, the above explained methodology was used to determine the weights of criterion as shown in Table 6 and these weights are integrated with VIKOR optimization process to obtain the final classification results.

IV. THE VIKOR TECHNIQUE (VISE KRITERIJUMSKA OPTIMIZACIJA KOMPROMISNO RESENJE)

The VIKOR method was first introduced and developed by Zeleny, in 1982 [22], as a one applicable technique for multi criteria optimization of complex systems based on the particular measure of “closeness” to the “ideal” solution. It determines the compromise ranking-list, the compromise solution, and the weight stability intervals for preference stability of the compromise solution obtained with the initial (given) weights [24]. It is suitable for situations in which the decision maker wants to achieve maximum profit and the risk of the decision is less important [21]. This method mainly focuses on ranking and selecting from a set of alternatives, and determines compromise solution for a problem with conflicting criteria, which can help the decision makers to reach a final solution [20].

Basically it is an aggregated statistical procedure to find the solution close to ideal and negative ideal solution. The best optimal solution is the result corresponding to smallest VIKOR indexed value. To calculate the VIKOR Index the following steps are used as per the existing research method of Rajesh Kumar Bhuyan et al 2016.

Step 1: Representation of normalized decision matrix,

The Normalized matrix may be defined as follows:

$$F = [f_{ij}]_{m \times n} \tag{8}$$

Here, $f_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$, $i = 1, 2, \dots, n$; and x_{ij} is the performance of alternative A_i with respect to the j^{th} criterion.

Step 2: Determination of ideal and negative-ideal solutions

The ideal (best) solutions A^* and the negative (worst) ideal solutions A^- are determined as follows:

$$A^* = \{(\max f_{ij} | j \in J) \text{ or } (\min f_{ij} | j \in J'), i = 1, 2, \dots, m\} \tag{9}$$

$$A^- = \{(\min f_{ij} | j \in J) \text{ or } (\max f_{ij} | j \in J'), i = 1, 2, \dots, m\} \tag{10}$$

Where, $J = \{j = 1, 2, \dots, n | f_{ij}, \text{ if desire response is large}\}$

$J' = \{j = 1, 2, \dots, n | f_{ij}, \text{ if desire response is small}\}$

In the present study, bulk density and porosity properties will be minimum value for ideal solutions and similarly maximum for negative ideal solution. In case of tensile strength and hardness properties, the ideal solution will be maximum value and the negative ideal solution will be minimum value from their correspondent column.

Step 3: calculation of utility measures and regret measures

The utility measure and the regret measure for each alternative can be calculated by following equations:

$$S_i = \sum_{j=1}^n W_j \frac{(f_j^* - f_{ij})}{(f_j^* - f_j^-)} \tag{11}$$

$$R_i = \text{Max}_j \left[W_j \frac{(f_j^* - f_{ij})}{(f_j^* - f_j^-)} \right] \tag{12}$$

Where, S_i and R_i , represent the utility measure and the regret measure, respectively, and W_j is the weight of the j^{th} criterion.

Step 4: Computation of VIKOR index

Calculation of VIKOR index of the i^{th} experimental run was obtained by substituting S_i and R_i into Eq. 12, yields the VIKOR index of the i^{th} experimental run as tabulated in Table 7.

$$Q_i = v \left[\frac{S_i - S^*}{S^- - S^*} \right] + (1 - v) \left[\frac{R_i - R^*}{R^- - R^*} \right] \tag{13}$$

Where, Q_i , represents the i^{th} alternative VIKOR value or multi performance characteristics index (MPCI) of i^{th} alternative, $i = 1, 2, \dots, m$; and v is the weight of the maximum group utility, usually it is to be set to 0.5.

$$S^- = \text{Max}_i (S_i); S^* = \text{Min}_i (S_i); R^- = \text{Max}_i (R_i); R^* = \text{Min}_i (R_i)$$

Step 5: Ranking the order

To rank the alternatives, smallest VIKOR index value is consider as the best solution and is the highest rank order. This smaller VIKOR index value produces the better multi-response performance index.

Step 6: Propose as a compromise solution of the consider weights of the given alternative. The alternative A^1 is considered as first highest rank by the measure Q (minimum) and A^2 is the second highest in the ranking order of the VIKOR index list.

If the following two condition are satisfied

a. *Acceptable advantage*

$$Q(A^2) - Q(A^1) \geq DQ = \frac{1}{(m-1)} \tag{14}$$

Where, m is the number of alternatives

b. *Acceptable stability in decision making*

Alternative A^1 is the best ranked by S or/and R . This compromise solution is stable within the decision making process, which could be the strategy of maximum group utility (when $v > 0.5$ is needed), or “by consensus” (when $v \approx 0.5$) or “with veto” (when $v < 0.5$). If one of the conditions is not satisfied, then the set of compromise solutions is proposed by,

This consists of:

I. the Alternatives A^1 and A^2 if only second (b) condition is not satisfied

II. Alternatives A^1, A^2, \dots, A^n first (a) condition is not satisfied A^n is calculated by the following relation

$$Q(A^n) - Q(A^1) < DQ, \text{ for maximum } n$$

Step 7: From the above the best rank when the Q value is minimum. The main objective is rank the list of experimental result and compromises the solution with their advantage rate. A smaller the VIKOR Index is the better result for the multi responses problem.

V. RESULTS AND DISCUSSION

The present study is to optimize the AHMMCs in terms of their mechanical and physical characteristics by integrating the criterion weights, determined by FAHP method, with VIKOR method. In order to optimize the AHMMCs the criterion weights are determined using FAHP method as explained in section 3 and these weights or priority vectors for each criterion are shown in below table (Table 6).

Table 6 Pair wise comparison matrix with linguistic ratings

Comparative judgments of the weights of the criteria with linguistic terms				
Factors	ρ_{bd}	σ_{ts}	HB	ϵ
ρ_{bd}	1	EqP	VStP	AP
σ_{ts}	EqP ⁻¹	1	FP	SP
HB	VStP ⁻¹	FP ⁻¹	1	EqP
ϵ	AP ⁻¹	SP ⁻¹	EqP ⁻¹	1
Comparative judgments of the criteria with fuzzy triangular numbers				
Factors	ρ_{bd}	σ_{ts}	HB	ϵ
ρ_{bd}	1	(1, 1, 2)	(6, 7, 8)	(8, 9, 10)
σ_{ts}	(1, 1, 2) ⁻¹	1	(4, 5, 6)	(2, 3, 4)
HB	(6, 7, 8) ⁻¹	(4, 5, 6) ⁻¹	1	(1, 1, 2)
ϵ	(8, 9, 10) ⁻¹	(2, 3, 4) ⁻¹	(1, 1, 2) ⁻¹	1
comparison judgments of α -cut fuzzy matrix				
Factors	ρ_{bd}	σ_{ts}	BHN	ϵ
ρ_{bd}	1	[1, 2]	[6, 8]	[8, 10]

σ_{ts}	[1/2, 1/1]	1	[4, 6]	[2, 4]	
HB	[1/8, 1/6]	[1/6, 1/4]	1	[1, 2]	
ϵ	[1/10, 1/8]	[1/4, 1/2]	[1/2, 1/1]	1	
Normalized fuzzy matrix					
Factors	ρ_{bd}	σ_{ts}	BHN	ϵ	Priority Vectors
ρ_{bd}	0.5	0.48	5.09	6.20	3.06
σ_{ts}	0.37	0.32	3.63	2.06	1.59
HB	0.07	0.06	0.72	1.03	0.47
ϵ	0.05	0.121	0.54	0.68	0.347
Lambda Max. (λ_{max})		4.2275			
Consistency Index (CI)		0.0758	n=4		
Consistency ratio (CR)		0.0842			

After calculating the criterion weights, the very first step in VIKOR method is to normalize the experimental results (Table 3) using Eq. 8. After normalization, need to calculate the ideal and negative ideal solution for each output response. Now the individual normalized decision matrix is analyzed by using Eq. (9) and Eq. (10) for determining the ideal (best) solution and negative (worst) ideal solution. For the responses such as bulk density (ρ_{bd}), tensile strength (σ_{ts}), hardness (BHN) and porosity (ϵ), the ideal values are 0.228971, 0.275659, 0.367402 and 0.141633 and the negative ideal values are 0.244756, 0.164679, 0.143959 and 0.341983, respectively. After determining the ideal and negative ideal solutions, need to calculate the utility and regret measures. The criterion weights in Table 6 are substituted in Eq. (11) and Eq. (12) for determining the utility measure and Regret measure. The maximum (S^-), minimum (S^*) utility measure are 4.956363, 0.176774 and maximum (R^-), minimum (R^*) regret measures are 3.0599997, 0.15272442, respectively. After calculating the values of Utility measure and Regret measure, will need to find the VIKOR index based on Eq. (13) for determination of optimal AHMMCs. The multi-criteria performance scores for each alternative can be determined from the VIKOR index. The best one is finally determined, in view of the fact that a smaller VIKOR value indicates an optimum composite. The VIKOR index Values are shown in Table 7, for each alternative.

Table 7 VIKOR Index (Q_i) and its rank order

Alternatives	Designations	VIKOR INDEX	Rank
1	A1	0.144740523	4
2	A2	0.077029705	2
3	A3	0.00715237	1
4	B1	0.192661658	7
5	B2	0.17287744	6
6	B3	0.153262339	5
7	C1	0.327666911	9
8	C2	0.2377214	8
9	C3	0.139521928	3
10	D1	0.383890985	12
11	D2	0.340456642	11
12	D3	0.335528522	10
13	E1	0.93861742	16
14	E2	0.827273973	14
15	E3	0.714454331	13
16	F1	1	18
17	F2	0.946656476	17
18	F3	0.874002111	15

The last step is to check the stable position and advantage over other experimental result, the acceptable advantage condition is applied by using the Eq. (14) as on step 6. From the above Table 7, it is observed that $Q(A^1)$ and $Q(A^2)$ are the first Index and the second Index, respectively. The corresponding VIKOR index values are 0 and 0.070585264 respectively. As per the first acceptable condition the values of $DQ = \frac{1}{(m-1)}$ is 0.058823529. Therefore $Q(A^2) - Q(A^1) = (0.07058264 - 0 = 0.07058264)$ is always greater than DQ and condition is acceptable.

VI. MICROSTRUCTURE ANALYSIS

The microstructural analysis of the matrix alloy revealed the presence of principal alloying elements. Figure 2 (a) shows the indistinguishable eutectic magnesium elements with other constituent of eutectic equilibrium precipitates of Mg /Al alloy along the grain boundaries.

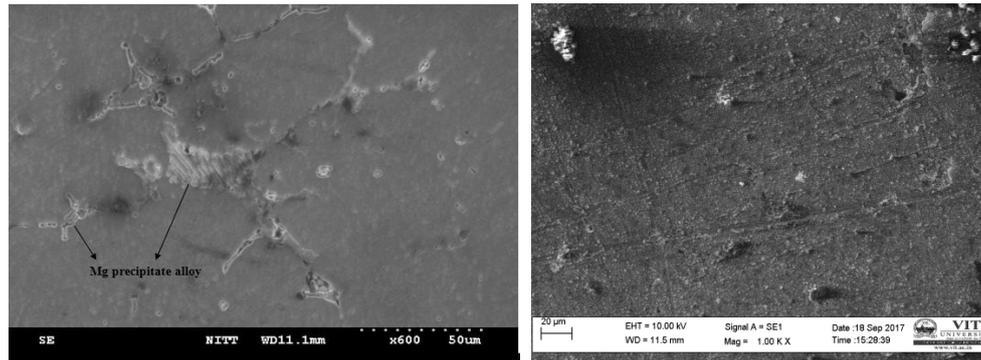


Figure 2 Micrographs of (a) AA 5083 matrix alloy and (b) A3 composite

From the FAHP-VIKOR analysis, A3 alternative was having the smallest VIKOR index value. Hence the A3 material is the optimum composite ie, 9% SiC along with flyash reinforced AA5083 composite [AA5083/9%(4.5%SiCp+4.5%FA)]. The SEM micrograph of obtained optimum AHMMC, fabricated with optimal conditions, is shown in figure 2 (b). As shown in Figure 2 (b), uniform distribution of hybrid particles was achieved with little (negligible) void.

VII. CONCLUSION

- In the present study, the material parameters such as matrix alloys (AA5083, AA6082 and AA7075), reinforcement materials (SiC along with flyash and Al₂O₃ along with flyash) and weight percent of reinforcement materials (3%, 6% and 9%) have been optimized for obtaining superior mechanical and better physical properties aluminium hybrid metal matrix composites.
- From the observation of Table 3, it is conclude that the increase of reinforcement weight percent in the matrix alloys increases the tensile strength and hardness of aluminium hybrid metal matrix composites whereas the density (measured density) decreases with increase of reinforcement percent.
- In order to select the optimum aluminium hybrid metal matrix composite in terms of stronger, harder, lower in density and porosity properties fuzzy AHP integrated VIKOR method has been used.
- After analyzing the data of AHMMCs with fuzzy AHP-VIKOR method, it is concluded that 9% wt. percent of SiC along with flyash reinforced AA5083 composite possess the superior mechanical and better physical properties.
- It is also concluded that SiC along with flyash reinforcement material possess the capabilities to improves the properties of matrix alloys.
- Finally from the results, it is concluded that fuzzy AHP-VIKOR method is effectively optimized the material factors for obtaining the superior metallurgical properties.
- Hence this optimization method is very much useful in prevention of vagueness and uncertainty in multi criteria decision making problems.
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