A STUDY ON CCS FOR GRAPHS THAT ALLOW JOHAN COLORING

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Abstract: A rainbow neighborhood of graph G is defined as the N[v] closed neighborhood of a chromatic coloring C of the C of G vertex v ∈ V(G) which has at least one vertex color of each color. In this work we found CCS from graphs including graph joins and path graphs, cycles and complete graphs admitting Johan Coloring, respectively. Ideas for future research are proposed.

Keywords: CCS, Johan Coloring, Johan Number, Rainbow Neighborhood, Product Graphs

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I. INTRODUCTION

For general notation and concepts in graphs and digraphs see [1, 3, 7, 8]. We will write that a graph G has order $\nu(G)=n \geq 1$ and size $\varepsilon(G) = p \geq 0$ with minimum and maximum degree $\delta(G)$ and $\Delta(G)$, respectively. The degree of a vertex $v \in V(G)$ is denoted $d_G(v)$ or when the context is clear, simply as $d(v)$. Also, without distinction, the vertices and the edges of a graph are called the elements of a graph. Unless mentioned otherwise all graphs G are finite, undirected simple graphs. We recall that if $C = \{c_1, c_2, c_3, ..., c_l\}$ and l sufficiently large, is a set of distinct colors, a proper vertex coloring of a graph G denoted $\phi: V(G) \rightarrowtail C$ is a vertex coloring such that no two distinct adjacent vertices have the same color. The cardinality of a minimum set of colors which allows a proper vertex coloring of G is called the chromatic number of G and is denoted $\chi(G)$.

In this study, we derived CCS from graphs including graph joining and the product of path graphs, cycles, and complete graphs that admit Johan Coloring. Definitions and findings on product graphs can be found in [2]. Future research ideas are offered.

II. REVIEW OF LITERATURE

First, the CCS was published in our article [4]. The CCS was defined and calculated for both classical graphs and graphs generated by certain operations. Prior to evaluating the CCS, these graphs were properly colored. A rainbow neighborhood of a graph G [6] with chromatic coloring C is defined as the closed neighborhood N[v] of a vertex $v \in V(G)$ that contains at least one colored vertex of each color in the chromatic coloring C of G. As a result of this research, a new graph coloring, known as Johan coloring, is considered to graphs as follows.

Definition 2.1. [4] For a finite, undirected simple graph G of order $\nu(G) = n \geq 1$ a CCS of G is a smallest induced subgraph H (smallest in respect of size of H) such that, the chromatic numbers of G and H are same.

Definition 2.2. [7] If C is the maximal coloring such that every vertex of G belongs to a rainbow neighborhood of G, it is referred to as the Johan coloring or the J-coloring of G. A graph G is J-colored if J-coloring is possible. The maximum number of colors in a J-coloring of a graph G, represented by J(G), is the J-coloring number of G.

III. JOHAN PROPERTIES AND THE CCS OF SOME GRAPHS

The section starts with classical results and some minor observations. Only finite, undirected connected simple graphs will be considered unless otherwise stated.

Proposition 3.1.

1. An acyclic graph G has J(G) = 2 and it has P_2 as a CCS.
2. An even cycle C_n, has J(G) = 2 and it has P_2 as a CCS.
3. The complete graph K_n, has J(G) = n and it has K_n as its unique CCS.
4. For C_3 and Odd wheel W_n, has J(G) = 3 and it has C_3 as a CCS.

Proof.

Part (1): The chromatic core subgraph is P_2 as any acycle graph can be colored with two colors. Also, it is evident that J(G) is two based on the definition of Johan coloring.

Part (2): In this case for even cycles the order is even say, n since even cycles can also be colored using two colors the proof is similar as in part (1).

Part (3): A complete graph with n vertices is n-chromatic, because all its vertices are adjacent. That is the chromatic number of complete graph K_n is n. So, $\chi(Kn)$ =n proof is obvious by the definition of Complete graph and Johan coloring.

Part (4): Note that as C_3 and K_3 are same, the chromatic core subgraphs are also same. Hence by part (3), $J(C_3)$ =$J(K_3)$.

In the case of Odd Wheel, we utilize a technique to color the wheel where the initial color is attached to the center of the wheel. The colors 2,3 are allocated in an alternating pattern to the vertices of the cycle. This coloring begins on any triangle face of the wheel and proceeds in a clockwise direction from the vertices of the cycle to the last vertex of the cycle. It is self-evident that any odd wheel can be three-colored and at least one C_3 exists. As a result, the CCS of any Odd Wheel is C_3. J(G)=3 as well.

Corollary 3.2. For odd cycle C_n, $n \geq 4$, and even wheel W_n, $n > 4$, the following results hold:

1. The Johan coloring does not exist.
2. Johan number does not exist.

Proof. In case of G be C_n, $n \geq 3$ or W_n, for even $n > 4$ the number of vertices is odd and all the vertices do not belong to rainbow neighborhood of G. It is therefore apparent that there is no Johan coloring either, and there is no Johan number.

Note: The above corollary holds true for graphs with pendent vertices.

3.1 CCS of Join of Graphs

The outcome of joining some graphs is the primary part of this subsection. By making a replica of graphs G and H and connecting the edges of the complete bipartite graph between the vertices V(G) and V(H), the join of two graphs G and H, denoted as G + H, is obtained. Note that only finite, undirected, connected simple graphs are examined unless otherwise specified.

Theorem 3.3. For graphs G and H with CCSs G′ and H′ respectively, the CCS of G + H is G′ + H′. For G and H can be C_3, even cycle, path, complete graphs.

Proof. Let $V(G) = \{v_1, \ldots, v_m\}$and $V(H) = \{v_1, v_2, \ldots, v_n\}$ then G + H is a complete bipartite graph. There is an edge between every vertex of G to every vertex of H and vice versa also occurs. Proposition 3.1. gives the CCS G′ and H′. On observing the graphs G + H and G′ + H′ we conclude the result that CCS of G + H is G′ + H′.

The following Corollary behaves different to the Theorem 3.4.

Remark: The CCS of $P_n + C_3$ is $P_n + C_3$.

3.2 CCS of Corona Product of Graphs

The following results apply to the corona graph $G \circ H$. Remember that $G \circ H$ is obtained by making one copy of graph G and $\nu(G) = n$ copies of graph H, and then constructing v + H for each vertex v in G.

Theorem 3.4. Let G and H be two graphs the CCS of G ∘ H is C_3 where G and H can be a path or an even cycle.

Proof. Let G and H be any graph which can be a path or an even cycle. Let V(G)= (v_1, v_2, …, v_n) and V(H)= (u_1, u_2, …, u_m), For each vertex v_i of G, we take a copy of H say H_i. The corona graph G ∘ H admits Johan coloring and its chromatic number is $\chi = 3$. The CCS of G ∘ H is C_3, for any path or an even cycle.

Theorem 3.5. Let K_m be a complete graph of m vertices and G be a graph which can be a path or an even cycle. The CCS of $K_m \circ G$ is given by,

$$(K_m \circ G) = \begin{cases} K_3 \; if \; m \leq 2 \\ K_m \; if \; m > 2 \end{cases}$$

Proof. Let K_m be a complete graph and G be a path or even cycle.

When m > 2 Let V(K_m) = (v_1, v_2, …, v_m) and V(G) = (u_1, u_2, …, u_n), For each vertex v_i of (K_m) we take a copy of G say G_i. The corona graph $K_m \circ G$ admits Johan coloring and its chromatic number is $\chi = m$. The CCS of $K_m \circ G$ is K_m, for any path or an even cycle form ≤ 2 the proof is similar.

Theorem 3.6. Let K_n be a complete graph with v vertices and G be any path or any even cycle. The CCS of G ∘ K_n is K_{n+1}

Proof. Let G be any path or any even cycle or n-complete graph and K_n be a complete graph. Let V(G)= (v_1, v_2, …, v_m) and V(K_n)= (u_1, u_2, …, u_n). For each vertex m_i of G, we take a copy of K say K_n. The corona graph G ∘ K_n admits Johan coloring and its chromatic number is $\chi = n+1$.The CCS of G ∘ K_n is K_{n+1}, for any path or an even cycle.

Theorem 3.7. Let K_m and K_n be a complete graph with m and n vertices respectively.
The CCS of $K_m \circ K_n$ is given by

$$(K_m \circ K_n) = \begin{cases} K_{n+1} & if \; m \leq 2 \\ K_{n+1} & if \; n \geq m-1 \; and \; m > 2 \\ K_m & if \; n < m-1 \; and \; m > 2 \end{cases}$$

Proof. Let K_m and K_n be both complete graphs.

Case 1: When m ≤ 2, Let V(K_m) = (v_1, v_2, …, v_m) and V(K_n) = (u_1, u_2, …, u_n). For each vertex m_i of K_m, we take a copy of K_n say K_n. The corona graph $K_m \circ K_n$ admits Johan coloring and its chromatic number is $\chi = n + 1$. The CCS of $K_m \circ K_n$ is K_{n+1}, for any complete graph.

Case 2: When m > 2 then the proof is similar.

IV. CONCLUSION

The concept of a CCS in relation to Johan coloring is introduced in this study. In this sector the notion of edge coloring can be generalized, and additional colorings such as local coloring, dynamic coloring, co-coloring, Grundy coloring, harmonious coloring, complete coloring, exact coloring, star coloring, etc., offer a wide range of applications.

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