A First Look at Eccentric Fuzzy Graphs

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Abstract : The objective of this paper is to define Eccentric membership function on the Vertex set and Edge set of the graph G and to construct Eccentric fuzzy graph using these functions. Also the order, size and degree of a vertex are defined for eccentric fuzzy graphs. Some bounds on eccentric membership function are established. In this paper, the concepts of operations on Eccentric fuzzy graphs are also derived.

***Index Terms* -** Eccentric membership function and Eccentric fuzzy graph.

I. INTRODUCTION

One of the notable Mathematical inventions of the 20th Century is that of Fuzzy sets by Lotfi. A. Zadeh [6] in 1965. He introduced the concepts of fuzzy subset of a set as a way for representing uncertainty. This idea have been applied to wide variety of scientific area. Research on the theory of fuzzy sets has been witnessing an exponential growth; both within Mathematics and in its Applications. This ranges from traditional mathematical subjects like Logic, Topology, Algebra, Analysis, etc to Pattern Recognition Theory, Artificial Intelligence, Operations Research, Neural Networks and Planning etc. Consequently, Fuzzy Set Theory has emerged as a potential area of Interdisciplinary Research and Fuzzy Graph Theory is of recent interest. Fuzzy graphs are useful to represent relationship which deal with uncertainty and it differs from Classical graph. The first definition of Fuzzy graph by Kaufman in 1965 was based on Zadeh Fuzzy relations. Rosenfeld [3] introduced another elaborate definition, including fuzzy vertex and fuzzy edges, several fuzzy analogues of graph theoretic concepts such as Paths, Cycles, Connectedness etc are also defined. Though the concept of fuzzy graph is very young, it has been growing fast and the numerous applications in various fields. The objective of this paper is to define Eccentric membership function on the Vertex set and Edge set of the graph G and to construct Eccentric fuzzy graph using these functions. Also the order, size and degree of a vertex are defined for eccentric fuzzy graph. Some bounds on eccentric membership function are established. In this paper, the concepts of operations on Eccentric fuzzy graphs are derived.

II. PRELIMINARIES

Graphs are simple models of relations. A graph is convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. Let G be a simple, finite and connected graph with vertex set V(G) and edge set E(G). The order p of the graph G is the number of vertices on the graph and the size q is the number of edges on the graph. The **distance** d(u, v) between the two vertices u and v of the graph G is the length (number of edges) of the shortest path between them. The **eccentricity** ecc(v) of a vertex v in a graph G is the distance from v to a vertex farthest from it, ecc(v) = max{d(u, v) / u ∈V(G)}. The **radius** of the graph G is defined as the minimum eccentricity of vertices in G and is denoted by rad(G). That is, rad(G) = min{ecc(u)/ u ∈V(G)}. The **diameter** of G is the maximum distance between two vertices of G and is denoted by diam(G). That is, diam(G) = max{ecc(u)/ u ∈V(G)}. For any v ∈ V(G) the **neighborhood** $N_G(v)$ (or simply N(v)) of v is the set of all vertices adjacent to v in G. The **degree** of a vertex v ∈V(G) is the number of edges incident with that vertex and is denoted by $deg_G(v)$ or deg(v). If all the vertices of a graph are of same degree, then the graph is a **regular graph**, otherwise it is an **irregular Graph**. A **cubic graph** is a regular graph in which all the vertices are of degree 3. For a **complete graph** K_p, all the vertices are of degree p – 1. A graph G is a **bipartite graph** if V(G) can be partitioned into two subsets U and W, called partite sets, such that every edge of G joins a vertex of U and a vertex of W. If every vertex of U is adjacent to every vertex of W, then G is called a **complete bipartite graph**. A complete bipartite graph with |U| = m and |W| = n is denoted by $K_{m,n}$.

When there is vagueness in the description of the objects or in its relationship or in both, it is natural to design a fuzzy graph model. Let V be a finite non-empty set and E be the collection of two element subset of V. A **Fuzzy Graph** F(G) = (σ, μ) is a set with a pair of membership functions, fuzzy vertex set function σ : V → [0, 1] and the fuzzy edge set function μ : E → [0, 1] such that μ(u, v) ≤ min{σ(u), σ (v)} (or σ(u) ∧ σ (v); ∧ stands for minimum) for all uv ∈ E(G). The **Underlying Crisp Graph** of the fuzzy graph F(G) = (σ, μ) is denoted by G* = (V, E), where V(G*) = {u ∈ V(G) : σ(u) > 0 } and E(G*) = {(u, v) ∈ V(G) ×V(G) : μ(u, v) > 0}. Let F(G) = (σ, μ) be a fuzzy graph on G = (V, E) and S ⊆ V(G) then the **order** p_f and **size** q_f of F(G) are defined as $p_f = \sum_{v\in V(G)} \sigma(v)$ and $q_f = \sum_{uv\in E(G)} \mu(u,v)$. An edge e = uv of a fuzzy graph is called an **effective edge** if μ(u, v)= min{σ(u),σ (v)}.

The **strength of the connectedness** between two nodes u, v in a fuzzy graph F(G) is $\mu^{\infty}(u,v)$ = sup { $\mu^{k}(u,v)$ / k = 1, 2, …}

where $\mu^k(u, v) \geq \sup\{\mu(u,u_1) \wedge \mu(u_1,u_2) \wedge \ldots \wedge \mu(u_{k-1},v)\}$. An arc (u, v) is said to be a **strong arc** $\mu(u, v) \geq \mu^{\infty}(u, v)$ and the node v is said to be the **strong neighbor** of u. If the arc (u, v) is not a strong arc then u is called **isolated node.** In a fuzzy graph F(G) every arc is strong arc then the fuzzy graph is called **strong arc fuzzy graph**. Let u be a node in fuzzy graph F(G) then N(u) = {v/ (u, v) is a strong arc} is called **neighborhood** of u and N[u] = N(u)∪{u} is called **closed neighborhood** of u. A fuzzy graph F(G) = (σ, μ) is said to be **connected** if any two vertices in G are connected.

III. MAIN RESULT

3.1. Eccentric Fuzzy Graph

Notation 3.1.1. $\left\lfloor \frac{a}{b} \right\rfloor$ denotes upto the first digit of the decimal when a is divided by b. For example $\left\lfloor \frac{2}{3} \right\rfloor = 0.6$ and $\left\lfloor \frac{5}{5} \right\rfloor = 1.0$.

Definition 3.1.2. Let G = (V, E) be a graph with diameter of G be diam(G). An **Eccentric Fuzzy Graph** EF(G) = (σ_e , μ_e) is a set with a pair of eccentric membership functions, eccentric fuzzy vertex set function $\sigma_e(G) : V(G) \to [0, 1]$ on the vertex set is defined as $\sigma_e(u) = \dfrac{ecc(u)}{diam(G)}$ for all u ∊ V(G) and the eccentric fuzzy edge set function $\mu_e(G) : E(G) \to [0, 1]$ on the edge set is defined as $\mu_e(u, v) = \min\{\sigma_e(u) , \sigma_e(v)\}$ for all uv ∊ E(G). That is, every edge is an effective edge.

Let EF(G) be an eccentric fuzzy graph on G(V, E). The **order** p_{ef} and **size** q_{ef} of the eccentric fuzzy graph EF(G)(σ_e, μ_e) are defined as $p_{ef} = \sum_{v \in V(ÆF(G))} \sigma_e(v)$ and $q_{ef} = \sum_{uv \in E(ÆF(G))} \mu_e(u, v)$ where v ∊ V(EF(G)) and uv ∊ E(EF(G)).

Example 3.1.3.

EF(G) ≅

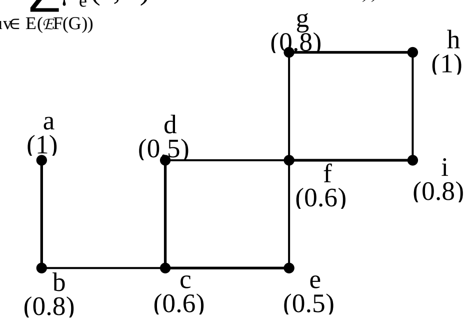


Figure3.1.4

ecc(a) = ecc(h) = 6; ecc(b) = ecc(i) = ecc(g) = 5; ecc(c) = ecc(f) = 4; ecc(d) = ecc(e) = 3.

diam(G) = max{ecc(u)/ u ∊ V(G) } = 6. For all v ∊ V(ÆF(G)), $\sigma_e(v) = \dfrac{ecc(v)}{diam(G)}$,

$\sigma_e(a) = \sigma_e(h) = \dfrac{6}{6} = 1$; $\sigma_e(b) = \sigma_e(i) = \sigma_e(g) = \dfrac{5}{6} = 0.8$; $\sigma_e(c) = \sigma_e(f) = \dfrac{4}{6} = 0.6$; $\sigma_e(d) = \sigma_e(e) = \dfrac{3}{6} = 0.5$.

Theorem 3.1.5. The eccentricity fuzzy graph EF(G) of order p_{ef} and size q_{ef} corresponding to the graph G(p, q), satisfies $p_{ef} \leq p$ and $q_{ef} \leq q$.

Proof. By the definition, $p_{ef} = \sum_{u \in V(ÆF(G))} \sigma_e(u) = \sum_{u \in V(ÆF(G))} \dfrac{ecc(u)}{diam(G)}$. The farthest path on the graph G with p vertices are of length p – 1.

Therefore $p_{ef} \leq \sum_{u \in V(ÆF(G))} \dfrac{p-1}{p-1}$

$\leq \sum_{u \in V(G)} 1$

$\leq 1 + 1 + \ldots + p$ times

$\leq p$.

$q_{ef} = \sum_{uv \in E(ÆF(G))} \mu_e(u, v)$

$= \sum_{uv \in E(ÆF(G))} \sigma_e(u) \wedge \sigma_e(v)$

$\leq \sum_{uv \in E(ÆF(G))} 1 \wedge 1$

$\leq \sum_{uv \in E(ÆF(G))} 1 \leq 1 + 1 + \ldots + q$ times

$\leq q$.

Hence $q_{ef} \leq q$.

This completes the proof of the theorem.

Theorem 3.1.6. Let EF(G) be an eccentric fuzzy graph of order p_{ef} and size q_{ef} corresponding to the graph G(p,q), then $p_{ef} = p$ and $q_{ef} = q$ if and only if ecc(u) = diam(G) for all u ∈ V(G).

Proof. Let $p_{ef} = p$ then by the definition of the eccentric fuzzy graph $\sum_{u \in V(\mathcal{E}F(G))} \sigma_e(u) = p$ implies $\sum_{u \in V(\mathcal{E}F(G))} \frac{ecc(u)}{diam(G)} = p$. This is possible only if ecc(u) = diam(G).

Conversely, let ecc(u) = diam(G) for all u ∈ V(G).

Then $p_{ef} = \sum_{u \in V(\mathcal{E}F(G))} \sigma_e(u)$

$= \sum_{u \in V(\mathcal{E}F(G))} \frac{ecc(u)}{diam(G)}$

$= \sum_{u \in V(\mathcal{E}F(G))} 1$

$= 1 + 1 + \ldots + p$ times

$= p$.

Also, let $q_{ef} = q$ then by the definition of the eccentric fuzzy graph $\sum_{uv \in E(\mathcal{E}F(G))} \mu_e(u,v) = q$ implies $\sum_{uv \in E(\mathcal{E}F(G))} \sigma_e(u) \wedge \sigma_e(v) = q$. This is possible only if ecc(u) = diam(G).

Conversely, let ecc(u) = diam(G) for all u ∈ V(G).

Then $q_{ef} = \sum_{uv \in E(\mathcal{E}F(G))} \mu_e(u,v)$

$= \sum_{uv \in E(\mathcal{E}F(G))} \sigma_e(u) \wedge \sigma_e(v)$

$= \sum_{uv \in E(\mathcal{E}F(G))} 1 \wedge 1$

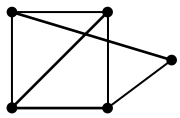
$= \sum_{uv \in E(\mathcal{E}F(G))} 1$

$= 1 + 1 + \ldots + q$ times $= q$.

Remark 3.1.7. (i). For a complete graph K_p, $p_{ef} = p$ and $q_{ef} = q$. In this case ecc(u) = 1 for all u ∈V(G).

(ii). For a complete bipartite graph $K_{m,n}$, $p_{ef} = p$ and $q_{ef} = q$. In this case ecc(u) = 2 for all u ∈V(G).

(iii). There exist an irregular graph in which $p_{ef} = p$ and $q_{ef} = q$. Consider the graph G, given in Figure 3.1.8.

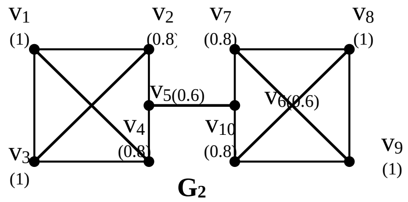


G_1

Figure 3.1.8.

For the graph G_1, ecc(u) = 2 for all u ∈ V(G_1). Therefore diam(G_1) = 2. Hence $p_{ef} = p = 5$ and $q_{ef} = q = 7$.

(iv). There exist a regular graph in which $p_{ef} < p$ and $q_{ef} < q$. Consider the graph G_2 given in Figure 3.1.9



G_2

Figure 3.1.9.

For the graph G_2,

$ecc(v_1) = ecc(v_3) = ecc(v_8) = ecc(v_9) = 5$; $ecc(v_2) = ecc(v_4) = ecc(v_7) = ecc(v_{10}) = 4$ and $ecc(v_5) = ecc(v_6) = 3$.

Also $diam(G_2) = \{max(ecc(u))/ u \in V(G_2)\} = 5$.

$$\sigma_e(v_1) = \frac{ecc(v_1)}{diam(G_2)} = \frac{5}{5} = 1.$$

Similarly $\sigma_e(v_3) = \sigma_e(v_8) = \sigma_e(v_9) = 1$;

$\sigma_e(v_2) = \frac{4}{5} = 0.8 = \sigma_e(v_4) = \sigma_e(v_7) = \sigma_e(v_{10})$;

$\sigma_e(v_5) = \frac{3}{5} = 0.6 = \sigma_e(v_6)$.

$p_{ef} = \sum_{u \in V(\mathcal{E}F(G_2))} \sigma_e(u)$

$= 4 \times 1 + 4 \times 0.8 + 2 \times 0.6$

$= 8.4 < 10 = p.$

$q_{ef} = \sum_{uv\in E(ÆF(G_2))} \mu_e(u,v)$

$= \sum_{uv\in E(ÆF(G_2))} \sigma_e(u) \wedge \sigma_e(v)$

$= 2\times (1+0.8+0.8+0.6+0.6+0.8+0.8) + 0.6$

$= 2 \times (5.4) + 0.6$

$= 11.5 < 15 = q.$

Theorem 3.1.10. Let EF(G) be an eccentric fuzzy graph corresponding to the graph G, then $\sigma_e(u) \geq \frac{1}{2}$ for all u∊V(G).

Proof. Let EF(G) be an eccentric fuzzy graph corresponding to the graph G. Then by the definition ecc(u) ≥ rad(G) for all u ∊ V(G). - - - - - (1).

Also diam(G) ≤ 2 rad(G). Hence $\frac{1}{\text{diam}(G)} \geq \frac{1}{2\times\text{rad}(G)}$ - - - - (2).

By the definition of eccentric membership function,

$$\sigma_e(u) = \frac{\text{ecc}(u)}{\text{diam}(G)} \geq \frac{\text{rad}(G)}{2\times\text{rad}(G)} \geq \frac{1}{2} \text{ (from (1) and (2)).}$$

Corollary 3.1.11. Let EF(G) be an eccentric fuzzy graph then $p_{ef} \geq \frac{p}{2}$ and $q_{ef} \geq \frac{q}{2}$.

Proof. Let EF(G) be an eccentric fuzzy graph corresponding to the graph G. Let p_{ef} and q_{ef} be the order and size of EF(G) corresponding to the graph G of order p and size q. By definition, $p_{ef} = \sum_{u\in V(ÆF(G))} \sigma_e(u) \geq \sum_{u\in V(ÆF(G))} \frac{1}{2}$ (by Theorem 3.1.10.) $\geq \frac{p}{2}$

$q_{ef} = \sum_{uv\in E(ÆF(G))} \mu_e(u,v)$

$= \sum_{uv\in E(ÆF(G))} \sigma_e(u) \wedge \sigma_e(v)$

$\geq \sum_{uv\in E(ÆF(G))} \frac{1}{2} \wedge \frac{1}{2}$

$\geq \sum_{uv\in E(ÆF(G))} \frac{1}{2} \geq \frac{q}{2}.$

Hence the Corollary.

Remark 3.1.12. Let EF(G) be an eccentric fuzzy graph then, from Theorem 3.1.5. and from Corollary 3.1.11. There is an immediate consequent that $\frac{p}{2} \leq p_{ef} \leq p$ and $\frac{q}{2} \leq q_{ef} \leq q$.

Definition 3.1.13. Let EF(G) be an Eccentric Fuzzy graph corresponding to the graph G then the **Degree of an Eccentric Fuzzy Graph** denoted by $\deg_{ef}(u)$ is defined by

$\deg_{ef}(u) = \deg(u) \times \sigma_e(u)$ for all u ∊ V(G).

Illustration 3.1.14. Consider a path on 5 vertices. That is G ≅ P_5. Then diam $(P_p) = 4$. Let $V(P_5) = \{v_1, v_2, v_3, v_4, v_5\}$ where v_1 and v_5 are terminal vertices.

Also $\sigma_e(v_1) = \frac{\text{ecc}(v_1)}{\text{diam}(P_5)} = \frac{4}{4} = 1 = \sigma_e(v_5)$;

$\sigma_e(v_2) = \frac{\text{ecc}(v_2)}{\text{diam}(P_5)} = \frac{3}{4} = 0.7 = \sigma_e(v_4)$;

$\sigma_e(v_3) = \frac{\text{ecc}(v_3)}{\text{diam}(P_5)} = \frac{2}{4} = 0.5.$

Hence $\deg_{ef}(v_1) = \deg(v_1) \times \sigma_e(v_1) = 1\times1= 1= \deg_{ef}(v_5)$;

$\deg_{ef}(v_2) = \deg(v_2) \times \sigma_e(v_2) = 2\times0.7 = 1.4 = \deg_{ef}(v_4)$

$\deg_{ef}(v_3) = \deg(v_3)\times\sigma_e(v_3) = 2\times0.5 = 1.$

Definition 3.1.15. Let EF(G) be an Eccentric Fuzzy graph corresponding to the graph G then the **Maximum Edge Membership Function** $\mu_e'(u,v) = \sigma_e(u) \vee \sigma_e(v)$ for all uv ∊E(EF(G)) where '∨' stands for the maximum value.

Illustration 3.1.16. Let G ≅ P_5 then

$\mu_e'(v_1,v_2) = \sigma_e(v_1) \vee \sigma_e(v_2) = 1 \vee 0.7 = 1 = \mu_e'(v_4,v_5)$;

$\mu_e'(v_2,v_3) = \sigma_e(v_2) \vee \sigma_e(v_3) = 0.7 \vee 0.5 = 0.7 = \mu_e'(v_3,v_4)$.

Proposition 3.1.17. Let EF(G) be an Eccentric Fuzzy graph corresponding to the graph G then, $\deg_{ef}(u) \leq \deg(u)$ for all u ∊ V(EF(G)).

Proof. By Definition 3.1.13., $\deg_{ef}(u) = \deg(u) \times \sigma_e(u)$ f or all u ∊ V(G).

But $\sigma_e(u) = \dfrac{ecc(u)}{diam(G)} \leq 1$.

Hence $deg_{ef}(u) \leq 1 \times deg(u) \leq deg(u)$ for all $u \in V(G)$.

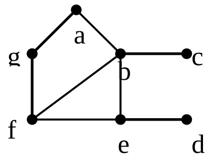
Definition 3.1.18. Let EF(G) be an Eccentric Fuzzy graph corresponding to the graph G then the **Maximum Size** q_{ef}' is defined as $q_{ef}' = \sum_{uv\in E(G)} \mu_e'(u, v)$.

From the definition it follows that $q_{ef} \leq q_{ef}'$.

Illustration 3.1.19. For the graph $G \cong P_5$, $q_{ef}' = \sum_{uv\in E(\mathcal{E}F(G))} \mu_e'(u, v) = 1 + 0.7 + 0.7 + 1 = 3.4$.

3.1.20. Handshaking Lemma: Let EF(G) be an Eccentric Fuzzy graph corresponding to the graph G, then the sum of the degrees of all the vertices of EF(G) is equal to twice the average of the size q_{ef} and maximum size q_{ef}' of EF(G).

Illustration 3.1.21.



ℰF(G)

Figure 3.1.22.

For the graph given in Figure 3.1.22.,

$ecc(a) = ecc(c) = ecc(d) = ecc(g) = 3$;
$ecc(b) = ecc(e) = ecc(f) = 2$ and
$diam(G) = 3$.

By definition $\sigma_e(a) = \dfrac{ecc(a)}{diam(G)} = \dfrac{3}{3} = 1 = \sigma_e(c) = \sigma_e(d) = \sigma_e(g)$;

$\sigma_e(b) = \dfrac{ecc(b)}{diam(G)} = \dfrac{2}{3} = 0.6 = \sigma_e(e) = \sigma_e(f)$.

By definition $deg_{ef}(a) = \sigma_e(a) \times deg(a) = 1\times 2 = 2$; $deg_{ef}(b) = 0.6\times 5 = 3.0$;
$deg_{ef}(c) = 1\times 1 = 1$; $deg_{ef}(d) = 1\times 1 = 1$;
$deg_{ef}(e) = 0.6 \times 3 = 1.8$; $deg_{ef}(f) = 0.6\times 3 = 1.8$;
$deg_{ef}(g) = 1\times 3 = 3$.

By definition $\mu_e(a,b) = \sigma_e(a) \wedge \sigma_e(b) = 1 \wedge 0.6 = 0.6 = \mu_e(g,f) = \mu_e(c,b) = \mu_e(d,e) = \mu_e(g,b)$;
$\mu_e(a,g) = 1 \wedge 1 = 1$; $\mu_e(b,e) = 0.6 \wedge 0.6 = 0.6 = \mu_e(b,f) = \mu_e(f,e)$.
$\mu_e'(a,b) = \sigma_e(a)\vee\sigma_e(b) = 1 \vee 0.6 = 1 = \mu_e'(g,f) = \mu_e'(c,b) = \mu_e'(d,e) = \mu_e'(g,b)$;
Similarly $\mu_e'(a,g) = 1$; $\mu_e'(b,e) = 0.6 = \mu_e'(b,f) = \mu_e'(f,e)$.
Sum of the degrees of all the vertices of EF(G)
$= deg_{ef}(a) + deg_{ef}(b) + deg_{ef}(c) + deg_{ef}(d) + deg_{ef}(e) + deg_{ef}(f) + deg_{ef}(g)$
$= 2 + 3 + 1 + 1 + 1.8 + 1.8 + 3 = 13.6$.
$q_{ef}(G) = \mu_e(a,b) + \mu_e(g,f) + \mu_e(c,b) + \mu_e(d,e) + \mu_e(g,b) + \mu_e(a,g) + \mu_e(b,e) + \mu_e(b,f) + \mu_e(f,e)$
$= 0.6 + 0.6 + 0.6 + 0.6 + 0.6 + 1 + 0.6 + 0.6 + 0.6 = 5.8$.
$q_{ef}'(G) = 1 + 1 + 1 + 1 + 1 + 1 + 0.6 + 0.6 + 0.6 = 7.8$.
$q_{ef}(G) + q_{ef}'(G) = 5.8 + 7.8 = 13.6$.

Twice the average of $q_{ef}(G)$ and $q_{ef}'(G) = 2\times \dfrac{q_{ef}(G) + q_{ef}'(G)}{2} = 13.6$. Hence the Lemma.

Definition 3.1.23. In an Eccentric Fuzzy graph $EF(G)(\sigma_e, \mu_e)$ the vertices which has the minimum vertex eccentric membership function are called the **central vertices**. The vertices which has the vertex eccentric membership function equal to 1 are called the **terminal vertices** and all the other vertices are the **intermediate vertices**.

Remark 3.1.24. There exists Eccentric Fuzzy graph EF(G) with no intermediate vertices.

Example 3.1.25. For the graph given in Figure 3.1.22., the vertices a, c, d and g are the terminal vertices and the vertices b, e and f are the central vertices. In this graph there are no intermediate vertices.

For the graph given in Figure 3.1.9., the vertices v_1, v_3, v_8 and v_9 are the terminal vertices and the vertices v_5 and v_6 are the central vertices and the vertices v_2, v_4, v_7 and v_{10} are the intermediate vertices.

3.2. Operations on Eccentric Fuzzy graph:

3.2.1.Union: Let $EF(G_1)$: $(\sigma_{e_1}, \mu_{e_1})$ and $EF(G_2)$: $(\sigma_{e_2}, \mu_{e_2})$ be two Eccentric Fuzzy graphs with $G_1^*(V_1,E_1)$ and $G_2^*(V_2,E_2)$. Let $G^* = G_1^* \cup G_2^* = (V_1\cup V_2, E_1\cup E_2)$ be the union of G_1^* and G_2^*. Then the union of two Eccentric Fuzzy Graphs $EF(G_1)$ and $EF(G_2)$ is a fuzzy graph

$G = G_1\cup G_2$: $(\sigma_{e_1} \cup \sigma_{e_2}, \mu_{e_1} \cup \mu_{e_2})$ defined by

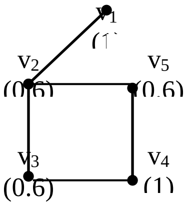
$$(\sigma_{e_1} \cup \sigma_{e_2})(u) = \begin{cases} \sigma_{e_1}(u); u\in V_1 - V_2 \\ \sigma_{e_2}(u); u\in V_2 - V_1 \\ \sigma_{e_1}(u) \wedge \sigma_{e_2}(u); u\in V_1 \cap V_2 \end{cases}$$

and

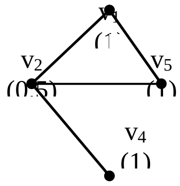
$$(\mu_{e_1} \cup \mu_{e_2})(u) = \begin{cases} \mu_{e_1}(u, v); uv\in E_1 - E_2 \\ \mu_{e_2}(u, v); u\in E_2 - E_1 \\ \mu_{e_1}(u) \wedge \mu_{e_2}(v); uv\in E_1 \cap E_2 \end{cases}$$

Remark 3.2.2 The union of two Eccentric Fuzzy Graphs EF(G_1) and EF(G_2) need not be an Eccentric Fuzzy Graph, but it is a Fuzzy Graph.

Example 3.2.3.

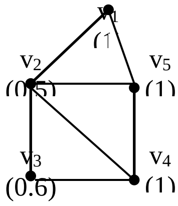


ƐF(G₁)
Figure 3.2.4.



ƐF(G₂)
Figure 3.2.5.

The union of two Eccentric Fuzzy Graphs EF(G_1) and EF(G_2) given in Figure 3.2.4. and Figure 3.2.5. is as follows



G
Figure 3.2.6.

The graph G given in Figure 3.2.6. is not an Eccentric Fuzzy Graph since $\sigma_e(v_3) \neq 1$.

3.2.7.Intersection: Let EF(G_1): $(\sigma_{e_1}, \mu_{e_1})$ and EF(G_2) : $(\sigma_{e_2}, \mu_{e_2})$ be two Eccentric Fuzzy graphs with $G_1^*(V_1,E_1)$ and $G_2^*(V_2,E_2)$. Let $G^* = G_1^* \cap G_2^* = (V_1 \cap V_2, E_1 \cap E_2)$ be the intersection of G_1^* and G_2^*. Then the intersection of two Eccentric Fuzzy Graphs EF(G_1) and EF(G_2) is a fuzzy graph $G = G_1 \cap G_2$: $(\sigma_{e_1} \cap \sigma_{e_2}, \mu_{e_1} \cap \mu_{e_2})$ defined by

$$(\sigma_{e_1} \cap \sigma_{e_2})(u) = \begin{cases} \sigma_{e_1}(u) \wedge \sigma_{e_2}(u); u \in V_1 \cap V_2 \\ 0; \text{otherwise} \end{cases} \text{ and } (\mu_{e_1} \cap \mu_{e_2})(u) = \begin{cases} \mu_{e_1}(u) \wedge \mu_{e_2}(v); uv \in E_1 \cap E_2 \\ 0; \text{otherwise} \end{cases}$$

Remark 3.2.8. The intersection of two Eccentric Fuzzy Graphs EF(G_1) and EF(G_2) need not be an Eccentric Fuzzy Graph, but it is a Fuzzy Graph.

Example 3.2.9. The intersection of two Eccentric Fuzzy Graphs EF(G_1) and EF(G_2) given in Figure 3.2.4 and Figure 3.2.5. is as follows

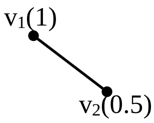


Figure 3.2.10.

The graph G given in Figure 3.2.10. is not an Eccentric Fuzzy Graph since $\sigma_e(v_2) \neq 1$.

3.2.11.Complement : Let EF(G): (σ_e, μ_e) be an Eccentric Fuzzy graph with G*(V,E). Then the complement of Eccentric Fuzzy Graph EF(G) is a fuzzy graph G^c: (σ_e^c, μ_e^c) defined by $\sigma_e^c(u) = \sigma_e(u)$ for all u ∊ V(EF(G)) and

$$\mu_e^c(u,v) = \begin{cases} 0 \text{ if } \mu_e(u,v) > 0 \text{ for } uv \in E(EF(G)) \text{ and} \\ \sigma_e(u) \wedge \sigma_e(u); \text{ otherwise.} \end{cases}$$

Remark 3.2.12. The complement of an Eccentric Fuzzy Graph EF(G) need not be an Eccentric Fuzzy Graph, but it is a Fuzzy Graph. It also follows that $((EF(G))^c)^c = EF(G)$.

Example 3.2.13. Consider the Eccentric Fuzzy Graph EF(G) given in Figure 3.2.14.

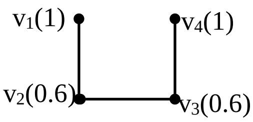


Figure 3.2.14.
ƐF(G)

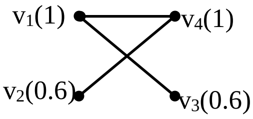


Figure 3.2.15.
$(ƐF(G))^c$

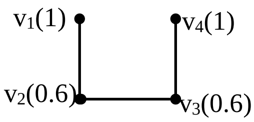


Figure 3.2.16.
$((ƐF(G))^c)^c$

Definition 3.2.17. An Eccentric Fuzzy graph EF(G) is **self complementary** if $(EF(G))^c = EF(G)$.

Example 3.2.18. Consider the Eccentric Fuzzy Graph EF(G) given in Figure 3.2.19.

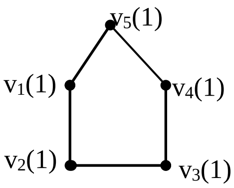


Figure 3.2.19
ƐF(G)

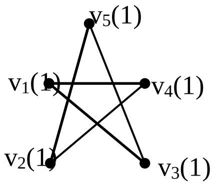


Figure 3.2.20.
(ƐF(G))^c

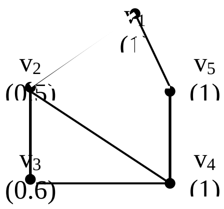
The complement of EF(G) is given in Figure 3.2.20 , since (EF(G))^c = EF(G) the Eccentric Fuzzy graph given in Figure 3. is a **self complementary** Eccentric Fuzzy graph.

3.2.21. Ring sum: Let EF(G_1): $(\sigma_{e_1}, \mu_{e_1})$ and EF(G_2) : $(\sigma_{e_2}, \mu_{e_2})$ be two Eccentric Fuzzy graphs with G_1*(V_1,E_1) and G_2*(V_2,E_2) . Let G* = G_1* ⊕ G_2* = (V_1∪V_2, (E_1∪E_2) – (E_1∩E_2)) be the ring sum of G_1* and G_2*. Then the ring sum of two Eccentric Fuzzy Graphs EF(G_1) and EF(G_2) is a fuzzy graph G = G_1⊕G_2: $(\sigma_{e_1} \oplus \sigma_{e_2}, \mu_{e_1} \oplus \mu_{e_2})$ defined by

$$(\sigma_{e_1} \oplus \sigma_{e_2})(u) = \begin{cases} \sigma_{e_1}(u); u \in V_1 - V_2 \\ \sigma_{e_2}(u); u \in V_2 - V_1 \\ \sigma_{e_1}(u) \wedge \sigma_{e_2}(u); u \in V_1 \cap V_2 \end{cases} \quad \text{and} \quad (\mu_{e_1} \oplus \mu_{e_2})(u) = \begin{cases} \mu_{e_1}(u,v); uv \in E_1 - E_2 \\ \mu_{e_2}(u,v); u \in E_2 - E_1 \\ \mu_{e_1}(u) \wedge \mu_{e_2}(v); uv \in E_1 \cap E_2 \end{cases}$$

Remark 3.2.22. The union of two Eccentric Fuzzy Graphs EF(G_1) and EF(G_2) need not be an Eccentric Fuzzy Graph, but it is a Fuzzy Graph.

Example 3.2.23. The ring sum of two Eccentric Fuzzy Graphs EF(G_1) and EF(G_2) given in Figure 3.2.4. and Figure 3.2.5. is as follows



G
Figure 3.2.24.

The graph G given in Figure 3.2.24. is not an Eccentric Fuzzy Graph since $\sigma_e(v_2) \neq 1$ and $\sigma_e(v_4) \neq 0.6$.

IV. CONCLUSION

In this paper the Eccentric Fuzzy Graph is defined and explained with illustrations. The operations on (crisp) graphs such as union, intersection, complement and ring sum are extended to Eccentric Fuzzy Graphs.

V. OPEN PROBLEMS

- To study the properties on the operations of Eccentric Fuzzy Graphs.
- To extend the operations of Join, Cartesian Product and Corona on Eccentric Fuzzy Graphs.

VI. ACKNOWLEDGMENT

The author assure that this paper is not presented in any conference or published in any other journals.

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