Fibonacci Sequence And It's Applications

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Abstract:
Fibonacci sequence of numbers and the associated "Golden Ratio" are manifested in nature and in certain works of art. We observe that many of the natural things follow the Fibonacci sequence. It appears in biological settings such as petals of sun flower, phyllotaxis (the arrangement of leaves on a stem) etc. At present Fibonacci numbers plays very important role in coding theory. Fibonacci numbers in different forms are widely applied in constructing security coding.

Keywords: Fibonacci Numbers, Golden ratio, Coding, Encryption, Decryption

Origin :

Leonardo Bonacci, known by most as Fibonacci, was arguably one of the most influential mathematicians in Europe in the 13th century. The book for which he is now famous, *Liber Abaci* (1202 A.D.) not only brought the Hindu- Arabic numeral system to the Western World, but also brought many interesting problems that had not been considered at the time.

Iteration	Pairs of Rabbits
1	1
2	1
3	2
4	3
5	5
6	8
7	13
8	21
9	34
⋮	⋮

Fibonacci's most recognized contribution to mathematics came in the form of one of these problems, which is now generally referred to as **The Rabbit Problem**. The problem is read as follows:

- Begin with one pair of [juvenile] rabbits.
- The rabbits must wait one iteration after birth to [mature and] begin to give birth.

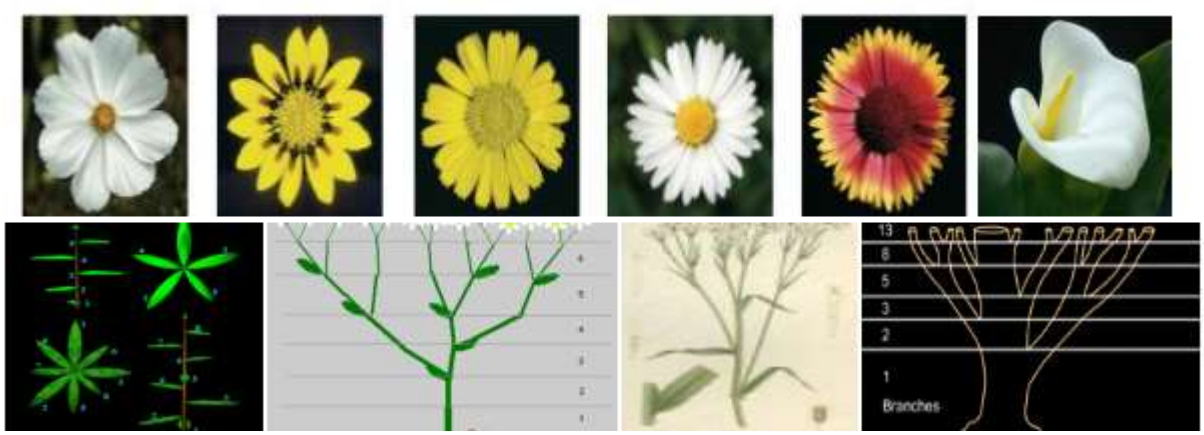
• Every iteration (after their first iteration of life), each pair of rabbits gives birth to one pair of rabbits that will also eventually reproduce.
• The rabbits live and reproduce indefinitely. So then, as can be seen in the table 1, a pattern forms for the total number of rabbits in a particular iteration. To find the current number of rabbits, one can take the sum of the previous two iterations'
number of rabbits! This is the origin of the now famous Fibonacci Sequence, with the first two numbers in the sequence being one (or equivalently zero and one
depending on the source).

Fibonacci Sequence In Nature

Fibonacci can be found in nature not only in the famous rabbit experiment, but also in beautiful flowers. On the head of a sunflower and the seeds are packed in a certain way so that they follow the pattern of the Fibonacci sequence. This spiral prevents the seed of the sunflower from crowding themselves out, thus helping them with survival. The petals of flowers and other plants may also be related to the Fibonacci sequence in the way that they create new petals.

Petals on flowers

Probably most of us have never taken the time to examine very carefully the number or arrangement of petals on
a flower. If we were to do so, we would find that the number of petals on a flower that still has all of its petals
intact and has not lost any, for many flowers is a Fibonacci number
• 1 petal: white cally lily
• 3 petals: lily, iris
• 5 petals: buttercup, wild rose, larkspur, columbine (aquilegia)
• 8 petals: delphiniums
• 13 petals: ragwort, corn marigold, cineraria,
• 21 petals: aster, black-eyed susan, chicory
• 34 petals: plantain, pyrethrum
• 55, 89 petals: michaelmas daisies, the asteraceae family



Plants show the Fibonacci numbers in the arrangements of their leaves .Three clockwise rotations, passing five leaves two counter-clockwise rotations. Sneezewort (Achillea ptarmica) also follows the Fibonacci numbers.

FIBONACCI SEQUENCE

In all above Example in nature, there is a number found by adding up the two numbers before it. Starting with 0 and 1, the sequence goes 0; 1; 1; 2; 3; 5; 8; 13; 21;34.and so forth. Written as a rule, the expression is

$$f_n = f_{n-1} + f_{n-2}$$

Math was incredibly important to those in the trading industry, and his passion for
numbers was cultivated in his youth Knowledge of numbers is said to have first originated in the Hindu-Arabic arithmetic system, which Fibonacci studied while growing up in North Africa. Prior to the publication of Liber abaci, the Latin-speaking world had yet to be introduced to the decimal number system. He wrote many books about geometry, commercial arithmetic and irrational numbers. He
also helped develop the concept of zero.

➢ Mathematical application :

As previously mentions, one can compute a Fibonacci number by using the previous two Fibonacci numbers as reference:

$$F_n = F_{n-1} + F_{n-2}.$$

However, in 1843 Binet derived a new formula for calculating the n^th^ Fibonacci number:

$$F_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}.$$

Fibonacci numbers have been applied in areas of combinatorics including a search algorithm that uses the Fibonacci numbers to find an element in a sorted array much like a binary search.

Fibonacci numbers also have many more applications as it can be applied to areas such as Pythagorean triples, or integers solutions to $a^2 + b^2 = c^2$, Pascal's triangle and many more.

The Fibonacci numbers also share a special bond with the golden ratio which is approximately $\phi \approx 1.618$ as the limit to infinity of the quotient of consecutive Fibonacci numbers converges to the golden ratio:

$$\lim_{n\to\infty} \frac{F_n}{F_{n-1}} = \phi.$$

GOLDEN RATIO

The golden ratio, given by $\lambda_1 = \frac{1+\sqrt{2}}{2}$; has been called many names throughout history. Some of these names include the golden number, golden proportion, golden mean, golden cut, golden section, divine proportion, the Fibonacci number and the mean of Phidias. Greek mathematicians defined it as the "division of a line in mean and extreme ratio" Since ancient times, the golden ratio has been of interest to many people in varying disciplines. The interest in the golden ratio goes back to at least 2600 B.C. when the Egyptians
were constructing the Great Pyramid. Theories suggest that the Egyptians were aware of the golden ratio and used it during the building of the Great Pyramid. Calculations show that the ratio between the base and hypotenuse of the right triangle inside the Great Pyramid is approximately 0.61762 which is very close to the reciprocal of the golden ratio. Thousands of years later in 1497, Italian mathematician Luca Pacioli wrote De Divina Proportione, which is thought to be the first book written about the golden ratio.

A Wonderful exampleof FIBONACCI SEQUENCE

Take any two consecutive numbers from this series as example 13 and 21 or 34 and 55. Now smaller number is in miles = the other one in Kilometer or bigger number is in Kilometers = the smaller one in Miles (The other way around).
34 Miles = round (54.72) Kilometers = 55 Kilometers
21 Kilometers = round (13.05) Miles = 13 Miles
For distances which are not exact Fibonacci values you can always proceed by splitting the distance into two or more Fibonacci values.

As example, for converting 15 km into miles we can proceed as following:
15 km = 13 km + 2 km

13 km -> 8 mile
2 km -> 1 mile
15 km -> 8+1 = 9 mile

Another example, for converting 170km into miles we can proceed as:
170 km = 10*17 km
17 km = 13 km + 2 km + 2 km = 8 + 1 + 1 miles = 10 miles (approximately)
Now, 170 km = 10*10 miles = 100 miles (approximately)
So, either way we can proceed. For bigger numbers we can proceed as above.

Fibonacci in Coding

Recently Fibonacci sequence and golden ratio are of great interest to the researchers in many fields of science including high energy physics, quantum mechanics, Cryptography and Coding. Raghu and Ravishankar(2015) developed a paper of application classical encryption techniques for securing data.(Raphael
and Sundaram,2012) showed that communication may be secured by the use of Fibonacci numbers. Similar application of Fibonacci in Cryptography is described here by a Simple Illustration. Suppose that Original Message"CODE" to be Encrypted. It is sent through an unsecured channel. Security key is chosen based on the Fibonacci number. Any one character may be chosen as a first security key to generate cipher text and then Fibonacci sequence can be used.

Method of Encryption

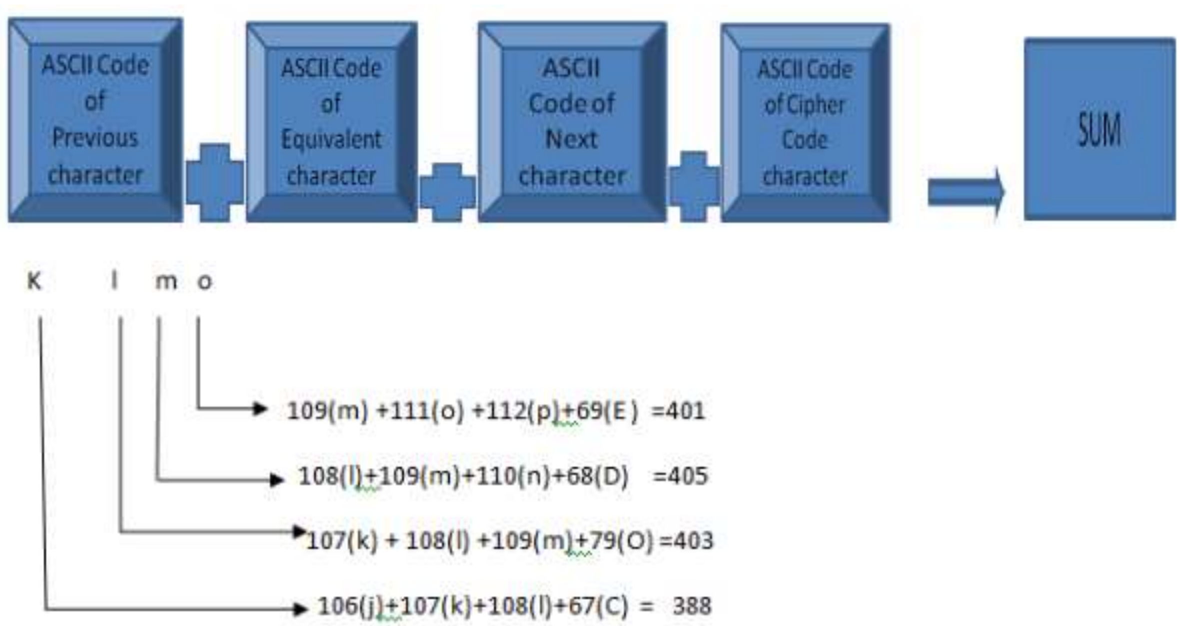
(a) For instance, let the first security key chosen be 'k'.

Plain Text: **C O D E**
Characters: **k l m o** p q r s t u v w x y z a b c d e f g h I j k l ….
Fibonacci : **1 2 3 5** ……..
Cipher Text: **k l m o**
Cipher Text is converted into Unicode symbols and saved in a text file. The text file is transmitted over the transmission medium. It is the first level of security.

(b) Cipher text to Unicode

In the second level of security, the ASCII code of each character obtained from the cipher text plus the ASCII code of its previous character, and next character is added to the ASCII code of the equivalent character in the original message. Here, ASCII codes of four characters are used as a security key to further encode the characters available in the cipher text to Unicode symbols.

For instance,



By looking at the symbols in a text file no unknown persons can identify what it is and the message cannot be retrieved unless the re-trivial procedure is known. Mukherjee and Samanta(2014) developed a paper where they used Fibonacci numbers in hiding image cryptography.

Decryption method

The Decryption process follows a reverse process of Encryption. Recipient extracted each symbol from the received text file and mapped to find its hexadecimal value .Obtained value is converted into a decimal value to find out the plain text using the key. Without knowledge of the key an unknown person cannot understand the existence of any secret message.

Conclusion

The Fibonacci numbers are Nature's numbering system. They appear everywhere in Nature, from the leaf arrangement in plants, to the pattern of the florets of a flower, the bracts of a pinecone, or the scales of a pineapple. The Fibonacci numbers are therefore applicable to the growth of every living thing, including a
single cell, a grain of wheat, a hive of bees, and even all of mankind. Nature follows the Fibonacci numbers astonishingly. But very little we observe the beauty of nature. The Great poet Rabindranath Tagore also noted this. If we study the pattern of various natural things minutely we observe that many of the natural things around us follow the Fibonacci numbers in real life which creates strange among us. The study of nature is very important for the learners. It increases the inquisitiveness

among the learners. The topic is chosen so that learners could be interested towards the study of nature around them. .Security in communication system is an interesting topic at present as India is going towards digitalization. A little bit of concept for securing data is also provided in this model. Let us finish by the words of Leonardo da Vinci "Learn how to see, Realize that everything connects to everything else".

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