RECURRENCE RELATIONS FOR THE GENERALIZED HYPERGEOMETRIC POLYNOMIAL SET $R_n(x_1, x_2, x_3)$

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Abstract: In this paper an attempt has been made to express some recurrence relations for the generalized hypergeometric polynomial set Rn(x_1,x_2,x_3) followed by important and interesting particular cases. Out of these particular results some of them stand for well known polynomials and some of them are believed to be new. Those recurrence relations are of at most important for mathematicians, scientists and engineers.

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1. INTRODUCTION

We defined the generalized hypergeometric polynomial set $R_n(x_1, x_2, x_3)$ by means of generating relation,

$$(1-\nu t)^{-\lambda}\left(1\text{-}\mu_1 x_2^{r_1} t^{r_1}\right)^{-\lambda_1}$$

$$F\left[\begin{matrix}(A_p);(C_u);(E_h)(G_m) \\ \qquad\qquad \mu x_1^{r_1} t, \mu_2 x_2^{-r_2} t^{r_2}, \mu_3 x_3^{-r_3} t^{r_3} \\ (B_q);(D_\nu);(F_k);(H_w)\end{matrix}\right]$$

$$= \sum_{n=0}^{\infty} R_{n,r;r_1;r_2;r_3;\quad (B_q);(D_\nu);(F_k);(H_w)}^{\nu;\lambda;\lambda_1;\lambda_2;\mu_1;\mu_2;\mu_3;(A_p);(C_u);(E_h);(G_m);}(x_1, x_2, x_3)\, t^n \qquad \ldots (1.1)$$

where ν, λ, λ_1, λ_2, λ_3 are real and r, r_1 are non-negative integer and r_2, r_3 are natural numbers.

The left hand side of (1.1) contains the product of generalized hypergeometric function and Lauricella function in the notation of Burchanall and Chaundy[1].

The polynomial set contains number of parameters, for simplicity we shall denote

$$R_{n,r;r_1;r_2;r_3;\quad (B_q);(D_\nu);(F_k);(H_w)}^{\nu;\lambda;\lambda_1;\lambda_2;\mu_1;\mu_2;\mu_3;(A_p);(C_u);(E_h);(G_m);}(x_1, x_2, x_3)$$

by $R_n(x_1, x_2, x_3)$.

where *n* denotes the order of the polynomial set.

After little simplification (1.1) gives

$$R_n(x_1,x_2,x_3)=\sum_{s=0}^{\left[\frac{n}{r}\right]}\sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]}\sum_{r_2=0}^{\left[\frac{n-r-r_1s_1}{e_2}\right]}\sum_{s_3=0}^{\left[\frac{n-r-r_1s_1-r_2s_2}{e_3}\right]}$$

$$\times\frac{\left[(A_p)\right]_{n-r-r_1s_1-(r_2-1)s_1-(r_3-1)s_3}}{\left[(B_q)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}}$$

$$\times\frac{\left[(C_u)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3}\left[(E_h)\right]_{s_2}\left[(G_m)\right]_{s_3}(\lambda)_s(\lambda_1)_{s_1}\,\nu^s\mu_1^{s_1}\mu_2^{s_2}\mu_3^{s_3}}{\left[(D_v)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3}\left[(F_k)\right]_{s_2}\left[(H_w)\right]_{s_3}\,s!s_1!s_2!s_3!}$$

$$\times\frac{\left(\mu x_1^{r_4}\right)^{n-r-r_1s_1-r_2s_2-r_3s_3}\,x_2^{r_1s_1+r_2s_2}}{(n-r-r_1s_1-r_2s_2-r_3s_3)!\,x_3^{r_3s_3}} \quad \ldots (1.2)$$

2. Notations

I. (i) $(n) = 1, 2, 3, \ldots\ldots\ldots n-1, n.$

(ii) $(a_p) = a_1, a_2, a_3, \ldots\ldots\ldots a_p.$

(iii) $(a_p\,;\,i) = a_1, a_2, \ldots\ldots a_{i-1}, a_{i+1} \ldots\ldots\ldots a_p.$

II. (i) $[(a_p)] = a_1\cdot a_2\cdot a_3, \ldots\ldots\ldots a_p.$

(ii) $\left[(a_p)\right]_n=\prod_{i=1}^{p}(a_i)_n=(a_1)_n(a_2)_n\ldots\ldots(a_p)_n.$

III. (i) $\Delta(a,b)=\frac{b}{a},\frac{b+1}{a},\ldots\ldots+\frac{b+a-1}{a}.$

(ii) $\Delta_k(a;b)=\left(\frac{b}{a}\right)_k\left(\frac{b+1}{a}\right)_k\ldots\ldots\left(\frac{b+a-1}{a}\right)_k$

$$=\prod_{r=1}^{a}\left(\frac{b+r-1}{a}\right)_k$$

(iii) $\Delta\left[m;(a_p)\right]=\prod_{i=1}^{p}\prod_{r=1}^{m}\left(\frac{a_i+r-1}{m}\right)_k.$

IV. (i) $\Gamma\left[(a_p)\right]=\prod_{i=1}^{p}\Gamma(a_i).$

(ii) $\Gamma\left[(a_p);s\right]=\prod_{i=s+1}^{p}\Gamma(a_i).$

(iii) $\Gamma\left[a+\frac{(m)}{m}\right]=\prod_{r=1}^{m}\Gamma\left(a+\frac{r}{m}\right).$

(iv) $\Gamma\left[\Delta(a;b)\right]=\prod_{r=1}^{a}\Gamma\left(\frac{b+r-1}{a}\right).$

V. (i) $\Gamma_*(a\pm b)=\Gamma(a+b)\Gamma(a-b).$

(ii) $\Gamma_*(a+b)=\Gamma(a+b)\Gamma(a-b).$

VI. (i) $M_1 = \frac{[(A_p)]_n [(C_u)]_n (\mu x_1^{r_4})^n}{[(B_q)]_n [(D_v)]_n n!}$

3. Recurrence Relations:

Now, we shall defined a few recurrence relations for the polynomial set $R_n(x_1, x_2, x_3)$.

From (1.2), we can write

$$R_n\left(x_1^{g_1}, x_2^{g_2}, x_3^{g_3}\right) = \sum_{s=0}^{\left[\frac{n}{r}\right]} \sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]} \sum_{s_2=0}^{\left[\frac{n-r-r_1s_1}{r_2}\right]} \sum_{s_3=0}^{\left[\frac{n-r-r_1s_1-r_2s_2}{r_3}\right]}$$

$$\times \frac{[(A_p)]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3} [(C_u)]_{n-r-r_1s_1-r_2s_2-r_3s_3} [(E_h)]_{s_2}}{[(B_q)]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3} [(D_v)]_{n-r-r_1s_1-r_2s_2-r_3s_3} [(F_k)]_{s_2}}$$

$$\times \frac{[(G_m)]_{s_3} (\lambda)_s (\lambda_1)_{s_1} \nu^s \mu_1^{s_1} \mu_2^{s_2} \mu_3^{s_3} x_2^{r_1s_1g_1+r_2s_2g_2} \mu^{n-r-r_1s_1-r_2s_2-r_3s_3} \left(x_1^{r4} g_1\right)^{n-r-r_1s_1-r_2s_2-r_3s_3}}{[(H_w)]_{s_3} s! s_1! s_2! x_3^{r_3s_3g_3} (n-r-r_1s_1-r_2s_2-r_3s_3)!} \quad \ldots (3.1)$$

Differentiating (3.1) with respect to x_1, we get

$$\frac{\partial}{\partial x_1}\left\{R_n\left(x_1^{g_1}, x_2^{g_2}, x_3^{g_3}\right)\right\} = g_1 r_4 \mu_1 \sum_{s=0}^{\left[\frac{n}{r}\right]} \sum_{s_1=0}^{\left[\frac{n-r}{r_1}\right]} \sum_{s_2=0}^{\left[\frac{n-r-r_1s_1}{r_2}\right]} \sum_{s_3=0}^{\left[\frac{n-r-r_1s_1-r_2s_2}{r_3}\right]}$$

$$\times \frac{[(A_p)]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3} [(C_u)]_{n-r-r_1s_1-r_2s_2-r_3s_3} [(E_h)]_{s_2}}{[(B_q)]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3} [(D_v)]_{n-r-r_1s_1-r_2s_2-r_3s_3} [(F_k)]_{s_2}}$$

$$\times \frac{[(G_m)]_{s_3} (\lambda)_s (\lambda_1)_{s_1} \nu^s \mu_1^{s_1} \mu_2^{s_2} \mu_3^{s_3} x_2^{r_1s_1g_1+r_2s_2g_2}}{[(H_w)]_{s_3} s! \quad s_1! \quad s_2! \quad x_3^{r_3s_3g_3}}$$

$$\times \frac{\mu^{n-r-r_1s_1-r_2s_2-r_3s_3} \left(x_1^{r4}\right)^{g_1(n-r-r_1s_1-r_2s_2-r_3s_3)-1}}{(n-r-r_1s_1-r_2s_2-r_3s_3)!} \quad \ldots (3.2)$$

$$R_{n,r;r_1;r_2;r_3;r_4;(B_q)+1;(D_v)+1;(F_k);(H_w)}^{\nu;\lambda;\lambda_1;\mu;\mu_1;\mu_2;\mu_3;(A_p)+1;(C_u)+1;(E_h);(G_m)}\left(x_1^{g_1}, x_2^{g_2}, x_3^{g_3}\right)$$

$$= g_1 r_4 x_1^{g_1 r_4 - 1} \mu_1 \sum_{s=0}^{\left[\frac{n-1}{r}\right]} \sum_{s_1=0}^{\left[\frac{n-1-r}{r_1}\right]} \sum_{s_2=0}^{\left[\frac{n-1-r-r_1s_1}{r_2}\right]} \sum_{s_3=0}^{\left[\frac{n-1-r-r_1s_1-r_2s_2}{r_3}\right]}$$

$$\times\frac{\left[\left(A_p\right)+1\right]_{n-1-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}\left[\left(C_u\right)+1\right]_{n-1-r-r_1s_1-r_2s_2-r_3s_3}}{\left[\left(B_q\right)+1\right]_{n-1-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}\left[\left(D_v\right)+1\right]_{n-1-r-r_1s_1-r_2s_2-r_3s_3}}$$

$$\times\frac{\left[\left(E_h\right)\right]_{s_2}\left[\left(G_m\right)\right]_{s_3}(\lambda)_s(\lambda_1)_{s_1}\nu^s\ \mu_1^{s_1}\mu_2^{s_2}\mu_3^{s_3}x_2^{r_1s_1+r_2s_2}}{\left[\left(F_k\right)\right]_{s_2}\left[\left(H_w\right)\right]_{s_3}s!\quad s_1!\quad s_2!\quad s_3!\quad x_3^{r_3s_3g_1}}$$

$$\times\frac{\mu^{n-1-r-r_1s_1-r_2s_2-r_3s_3}\left(x_1^{r_4}\right)^{g_1(n-1-r-r_1s_1-r_2s_2-r_3s_3)}}{(n-1-r-r_1s_1-r_2s_2-r_3s_3)!}$$

$$\therefore\ \frac{\left[\left(A_p\right)\right]\left[\left(C_u\right)\right]\mu x_1^{g_1r_4-1}}{\left[\left(B_q\right)\right]\left[\left(D_v\right)\right]}R_{n-1,\left(B_q\right)+1,\left(D_v\right)+1}^{\left(A_p\right)+1,\left(C_u\right)+1}\left(x_1^{g_1},x_2^{g_2},x_3^{g_3}\right)$$

$$=g_1r_4\sum_{s=0}^{\left[\frac{n-1}{r}\right]}\sum_{s_1=0}^{\left[\frac{n-1-r}{r_1}\right]}\sum_{s_2=0}^{\left[\frac{n-1-r-r_1s_1}{r_2}\right]}\sum_{s_3=0}^{\left[\frac{n-1-r-r_1s_1-r_2s_2}{r_3}\right]}$$

$$\times\frac{\left[\left(A_p\right)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}\left[\left(C_u\right)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3}\left[\left(E_h\right)\right]_{s_2}}{\left[\left(B_q\right)\right]_{n-r-r_1s_1-(r_2-1)s_2-(r_3-1)s_3}\left[\left(D_v\right)\right]_{n-r-r_1s_1-r_2s_2-r_3s_3}\left[\left(F_k\right)\right]_{s_2}}$$

$$\times\frac{\left[\left(G_m\right)\right]_{s_3}(\lambda)_s\nu^s(\lambda_1)_{s_1}\mu_1^{s_1}\ \mu_2^{s_2}\ \mu_3^{s_3}\ x_2^{r_1s_1g_1+r_2s_2g_2}}{\left[\left(H_w\right)\right]_{s_3}\ s!\quad s_1!\quad s_2!\quad s_3!\quad x_3^{r_3s_3g_3}}$$

$$\times\frac{\mu^{n-r-r_1s_1-r_2s_2-r_3s_3}\left(x_1^{r_4}\right)^{g_1(n-r-r_1s_1-r_2s_2-r_3s_3)}}{(n-1-r-r_1s_1-r_2s_2-r_3s_3)!}\qquad \ldots (3.3)$$

Hence from equation (3.2) and (3.3), we arrive at

$$\frac{\partial}{\partial x_1}\left\{R_n\left(x_1^{g_1},x_2^{g_2},x_3^{g_3}\right)\right\}=\frac{\left[\left(A_p\right)\right]\left[\left(C_u\right)\right]g_1r_4\mu\ x_1^{g_1r_4-1}}{\left[\left(B_q\right)\right]\left[\left(D_v\right)\right]}$$

$$\times R_{n-1,\left(B_q\right)+1,\left(D_v\right)+1}^{\left(A_p\right)+1,\left(C_u\right)+1}\left(x_1^{g_1},x_2^{g_2},x_3^{g_3}\right)\qquad \ldots (3.4)$$

where $n \geq 1$

Corollary : On setting $g_1 = 1 = g_2 = g_3$, we achieve

$$\frac{\partial}{\partial x_1}\left\{R_n\left(x_1,x_2x_3\right)\right\}=\frac{\left[\left(A_p\right)\right]\left[\left(C_u\right)\right]\mu r_4x_1^{r_4-1}}{\left[\left(B_q\right)\right]\left[\left(D_v\right)\right]}$$

$$\times R^{(A_p)+1,(C_u)+1}_{n-1,(B_q)+1,(D_v)+1}\left(x_1,x_2,x_3\right) \qquad \ldots (3.5)$$

where $n \geq 1$

Particular Cases of (3.5) :

I. On setting $p = 0 = q = u = v = s$; $\lambda = 1 = \nu = \lambda_1 = r = r_4$; $r_1 = 2 = \mu$; $\mu_1 = -4$ and writting x for x_1 in (3.5) we achieve

$$\frac{d}{dx}H_n(x) = 2nH_{n-1}(x)$$

where $H_n(x)$ are the Hermite Polynomials.

II. On making the snbstitution $p = 0 = q = u = h = s = r$; $r_2 = v = 1 = k = \lambda = \nu = \mu_2 = r_4$; $\mu = 1 = \mu_1$; $D_1 = 1 + \alpha$; $F_1 = 1 + \beta$; and in stead of $x_2 = \frac{x-1}{x+1}$ in (3.5), we get

$$(x-1)\frac{d}{dx}P_n^{(\alpha,\beta)}(x) = nP_n^{(\alpha,\beta)}(x) - (n+\alpha)P_{n-1}^{(\alpha,\beta+1)}(x)$$

where $P_n^{(\alpha,\beta)}(x)$ are the Jacobi Polynomials.

III. If we take $p = 0 = q = u = h = s$; $v = 1 = k = \lambda = \nu = x_2 = r_2 = r_4$; $\mu = \frac{1}{2} = \mu_1$; $D_1 = 1 + \beta$; $F_1 = 1 + \alpha$ and writing $\frac{x+1}{x-1}$ for x_1 in (3.5), we obtained

$$(x+1)\frac{d}{dx}P_n^{(\alpha,\beta)}(x) = nP_n^{(\alpha,\beta)}(x) + (n+\beta)P_{n-1}^{(\alpha+1,\beta)}(x)$$

where $P_n^{(\alpha,\beta)}(x)$ are the Jacobi Polynomials. [2, P.254]

IV. On taking $p = 0 = q = u = h = s$; $v = 0 = k = r = r_2 = r_4 = \lambda = \nu = x_2$; $D_1 = \lambda + \frac{1}{2} = F_1$; $\mu = \frac{1}{2} = \mu_2$ and writing $\frac{x+1}{x-1}$ for x_1 in (3.5), we get

$$(x^2-1)\frac{d}{dx}C_n^{(\lambda)}(x) = nxC_n^{(\lambda)}(x) - (2\lambda+n-1)C_{n-1}^{(\lambda)}(x)$$

where $C_n^{\lambda}(x)$ are the Gegenbauer Polynomials. [2, P.276]

V. On making the substitution $p = 0 = q = u = s$; $u = v = 1 = r = r_4 = x_2 = \lambda = \nu = \mu = \mu_2$; $F_1 = \lambda + \frac{1}{2}$, $r_2 = 2$ and writing $\frac{x}{\sqrt{x^2-1}}$ for x_1 in (3.5), we get

$$\frac{d}{dx}\left(C_n^{(\lambda)}(x)\right) = 2C_{n-1}^{\lambda+1}(x)$$

where $C_n^{\lambda}(x)$ are the Gegenbauer Polynomials. [2, P.276]

VI. If we put $p = 0 = q = u = v = m$; $w = 1 = r = r_4 = \nu = \lambda = \mu = \mu_3 = r_3$; $H_1 = 1$, $r_3 = 2$ and $\frac{x}{\sqrt{x^2-1}}$ for x_1 in (3.5), we get

$$(x^2-1)\frac{d}{dx}P_n(x) = n\left[xP_n(x) - P_{n-1}(x)\right]$$

where $P_n(x)$ are the Legendre Polynomials. [2, P.157]

Now equation (3.4) can be written as

$$\left(x_1^{1-g_1 r_4}\frac{\partial}{\partial x_1}\right) = R_n\left(x_1^{g_1}, x_2^{g_2}, x_3^{g_3}\right)$$

$$= \frac{\mu r_4 g_1\left[\left(A_p\right)\right]\left[\left(C_u\right)\right]}{\left[\left(B_q\right)\right]\left[\left(D_v\right)\right]} R_{\left(B_q\right)+1,\left(D_v\right)+1}^{\left(A_p\right)+1,\left(C_u\right)+1}\left(x_1^{g_1}, x_2^{g_2}, x_3^{g_3}\right) \quad \ldots (3.6)$$

where $n \geq 1$

Differentiating (3.6) successively *m*-times, we arrive at

$$\left(x_1^{1-r_4 g_1}\frac{\partial}{\partial x_1}\right)^m R_n\left(x_1^{g_1}, x_2^{g_2}, x_3^{g_3}\right)$$

$$= \frac{\left(\mu r_4 g_1\right)^m\left[\left(A_p\right)\right]_m\left[\left(C_u\right)\right]_m}{\left[\left(B_q\right)\right]_m\left[\left(D_v\right)\right]_m} R_{n-m,\left(B_q\right)+m,\left(D_v\right)+m}^{\left(A_p\right)+m,\left(C_u\right)+m}\left(x_1^{g_1}, x_2^{g_2}, x_3^{g_3}\right) \quad \ldots (3.7)$$

where $n \geq m$

Corollary : On setting $g_1 = 1 = g_2 = g_3$; we achieve

$$\left(x_1^{1-r_4}\frac{\partial}{\partial x_1}\right)^m R_n\left(x_1, x_2, x_3\right)$$

$$= \frac{\left(\mu r_4\right)^m\left[\left(A_p\right)\right]_m\left[\left(C_u\right)\right]_m}{\left[\left(B_q\right)\right]_m\left[\left(D_v\right)\right]_m} R_{n-m,\left(B_q\right)+m,\left(D_v\right)+m}^{\left(A_p\right)+m,\left(C_u\right)+m}\left(x_1, x_2, x_3\right) \quad \ldots (3.8)$$

where $n \geq m$ and *m* is a non-negative integer.

Particular Cases of (3.8) :

I. On setting $p = 0 = q = u = v = s$; $\lambda = 1 = \nu = \lambda_1 = r = r_4$; $r_1 = 2 = \mu$; $\mu_1 = -4$ and writting x for x_1 in (3.8) we achieve

$$\frac{d^m}{dx^m}H_n(x) = \frac{2^m n!}{(n-m)!}H_{n-m}(x)$$

where $H_n(x)$ are the Hermite Polynomials. [2, P.187]

II. On making the snbstitution $p = 0 = q = u = h = s = r$; $r_2 = v = 1 = k = \lambda = \nu = \mu_2 = r_4$; $\mu = 1 = \mu_1$; $D_1 = 1 + \alpha$; $F_1 = 1 + \beta$; and in stead of $x_2 = \frac{x-1}{x+1}$ in (3.8), we get

$$\frac{d^m}{dx^m}\left\{(x-1)^m P_n^{(\alpha,\beta)}\left(\frac{x+1}{x-1}\right)\right\} = \frac{(1+\alpha)_n (x-1)^{n-m}}{(1+\alpha)_{n-m}} P_{n-m}^{(\alpha,\beta+m)}\left(\frac{x-1}{x+1}\right)$$

where $P_n^{(\alpha,\beta)}(x)$ are the Jacobi Polynomials. [2, P.254]

III. If we take $p = 0 = q = u = h = s$; $v = 1 = k = \lambda = \nu = x_2 = r_2 = r_4$; $\mu = \frac{1}{2} = \mu_1$; $D_1 = 1 + \beta$; $F_1 = 1 + \alpha$ and writing $\frac{x+1}{x-1}$ for x_1 in (3.8), we obtained

$$\frac{d^m}{dx^m}\left\{(1-x)^n P_n^{(\alpha,\beta)}\left(\frac{1+x}{1-x}\right)\right\} = \frac{(1+\beta)_n (1-x)^{n-m}}{(1+\beta)_{n-m}} P_{n-m}^{(\alpha+m,\beta)}\left(\frac{1+x}{1-x}\right)$$

where $P_n^{(\alpha,\beta)}(x)$ are the Jacobi Polynomials.

IV. On taking $p = 0 = q = u = h = s$; $v = 0 = k = r = r_2 = r_4 = \lambda = \nu = x_2$; $D_1 = \lambda + \frac{1}{2} = F_1$; $\mu = \frac{1}{2} = \mu_2$ and writting $\frac{x+1}{x-1}$ for x_1 in (3.8), we get

$$\frac{d^m}{dx^m}\left\{(x^2-1)^{\frac{n}{2}} C_n^{(\lambda)}\left(\frac{x}{\sqrt{x^2-1}}\right)\right\} = \frac{(2\lambda)_n (x^2-1)^{\frac{n-m}{2}}}{(2\lambda)_{n-m}} C_{n-m}^{(\lambda)}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

where $C_n^{\lambda}(x)$ are the Gegenbauer Polynomials.

V. On making the snbstitution $p = 0 = q = u = s$; $u = v = 1 = r = r_4 = x_2 = \lambda = \nu = \mu = \mu_2$; $F_1 = \lambda + \frac{1}{2}$, $r_2 = 2$ and writting $\frac{x}{\sqrt{x^2-1}}$ for x_1 in (3.8), we get

$$\frac{d^m}{dx^m}\left\{(x-1)^n C_n^{\lambda}\left(\frac{x+1}{x-1}\right)\right\} = \frac{(2\lambda)_n x^{n-m}}{(n-m)!} {}_2F_1\left[\begin{matrix} -n+m, \frac{1}{2}-\lambda-n; & \\ & \frac{1}{\alpha} \\ \lambda+\frac{1}{2}; & \end{matrix}\right]$$

where $C_n^{\lambda}(x)$ are the Gegenbauer Polynomials.

VI. If we put $p = 0 = q = u = v = m$; $w = 1 = r = r_4 = \nu = \lambda = \mu = \mu_3 = r_3$; $H_1 = 1$, $r_3 = 2$ and $\frac{x}{\sqrt{x^2-1}}$ for x_1 in (3.8), we get

$$\frac{d^m}{dx^m}\left\{(x^2-1)^{\frac{n}{2}} P_n\left(\frac{x}{\sqrt{x^2-1}}\right)\right\} = \frac{n!(x^2-1)^{\frac{n-m}{2}}}{(n-m)!} P_{n-m}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

where $P_n(x)$ are the Legendre Polynomials.

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