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GENERALIZED SKEW DERIVATIONS IN SEMIPRIME RINGS

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Abstract: This presents some skew derivations in semiprime rings. Let R be a semiprime ring, $F: R \rightarrow R$ be a generalized skew derivation associated with skew derivation d and $H: R \rightarrow R$ be a left centralizer. If (i) $F(uv) \mp H(uv) = 0$ (ii) $F(uv) \mp H(vu) = 0$; (iii) $F(u)F(v) \mp H(uv) = 0$; (iv) $F(uv) \mp H(uv) \in C_{1,\alpha}$; (v) $F(uv) \mp H(vu) \in C_{1,\alpha}$; (vi) $F(u)F(v) \mp H(uv) \in C_{1,\alpha}$ for all $u, v \in R$.

Keywords: derivation, generalized derivation, left centralizer, semiprime ring, skew derivation and generalized skew derivation.

Introduction:

Bresar in [3], first time introduced the notion of generalized derivation. Daif et al. in [9], proved a result which is given as let R be a semiprime ring, I be a non zero ideal of R and d be a derivation on R such that $d([x,y]) = [x,y]$, for all $x, y \in I$, then $I \subseteq Z(R)$. In 2001, Ashraf et al. in [1], proved a result which is given as let R be a prime ring, I be a non zero two sided ideal of R and d be a derivation on R such that $d(xy) \pm xy \in Z(R)$, for all $x, y \in I$, then R is commutative. In 2003, Quadri et al. in [19] extended the result of Ashraf et al. in [2] on generalized derivation given as let R be a prime ring with characteristic different from two, I be a nonzero ideal of R and F be a generalized derivation on R associated with a derivation d on R such that $F([x,y]) = [x,y]$, for all $x, y \in I$, then R is commutative. In 2017, Didem K. Camci and Neset Aydin in [14] studied On Multiplicative (generalized)-derivation in semiprime rings. In 2000, C.L. Chung et al. and in [7] investigated on identities with skew derivations. In 2006, C.L. Chung et al. in [8] proved some results on skew derivations with annihilating Engel conditions. The skew derivations have been extensively studied by many researchers from various views (see for instance [13] and [18], where further references can be found). In 2004, T.K. Lee in [17] studied on generalized skew derivation characterized by acting on zero products. In 2006, H.W. Cheng et al. in [6] proved some results on Generalized skew derivations on rings. The generalized skew derivations have been extensively studied by many researchers from various views [4,5,10,11,12,15,16,20,21,22], motivated by the results in this paper we prove some results on generalized skew derivations in semiprime rings.

Preliminaries:

Throughout this paper R denote an associative ring with center $C_{1,\alpha}$. Recall that a ring R is semiprime if $xRx = \{0\}$ implies $x = 0$. For any $x, y \in R$, the symbol $[x, y]$ stands for the commutator $xy - yx$. The $(1, \alpha)$-centre of R denoted by $C_{1,\alpha}$ and defined by $C_{1,\alpha} = \{c \in R: cr = \alpha(r)c$, for all $r \in R\}$. An additive mapping $d: R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$. An additive mapping $d: R \to R$ is called a skew derivation if $d(xy) = d(x)y + \alpha(x)d(y)$, for all $x, y \in R$. An additive mapping $F: R \to R$ is called a generalized derivation, if there exists a derivation $d: R \to R$ such that $F(xy) = F(x)y + xd(y)$, for all $x, y \in R$. An additive mapping $F: R \to R$ is said to be a generalized skew derivation of R, if there exists a skew derivation $d: R \to R$ such that $F(xy) = F(x)y + \alpha(x)d(y)$, for all $x, y \in R$. An additive mapping $H: R \to R$ is called a left centralizer if $H(xy) = H(x)y$ for all $x, y \in R$, where α is automorphisms of R.

Throughout this paper, we shall make use of the basic commutator identities:

$[x, yz] = y[x, z] + [x, y]z$; $[xy, z] = [x, z]y + x[y, z]$; $[xy, z]_{1,\alpha} = x[y, z]_{1,\alpha} + [x, \alpha(z)]y$.

Lemma 1: Let R be a semiprime ring. If $F: R \to R$ is a generalized skew derivation associated with skew derivation d, then $d(uv) = d(u)v + \alpha(u)d(v)$, for all $u, v \in R$.

Proof:

We have $F(u(vw)) = F(u)vw + \alpha(u)d(vw)$, for all $u, v, w \in R$. (1)

On the other hand, we have $F\big((uv)w\big) = F(uv)w + \alpha(uv)d(w)$

$= F(u)vw + \alpha(u)d(v)w + \alpha(uv)d(w)$, for all $u, v, w \in R$. (2)

Equating equations (1) and equation (2), we get.

$F(u)vw + \alpha(u)d(vw) = F(u)vw + \alpha(u)d(v)w + \alpha(uv)d(w)$

$\alpha(u)\big(d(vw) - d(v)w - \alpha(v)d(w)\big) = 0$, for all $u, v, w \in R$. (3)

Left multiplying equation (3) by $d(vw) - d(v)w - \alpha(v)d(w)$, we get.

$(d(vw) - d(v)w - \alpha(v)d(w))\alpha(u)\big(d(vw) - d(v)w - \alpha(v)d(w)\big) = 0$, for all $u, v, w \in R$. Since α is an automorphism of R, we get.

$(d(vw) - d(v)w - \alpha(v)d(w))R\big(d(vw) - d(v)w - \alpha(v)d(w)\big) = 0$, for all $v, w \in R$. Since R is semiprime ring, we get.

$d(uv) = d(u)v + \alpha(u)d(v)$, for all $u, v \in R$. That is, d is a skew derivation.

Lemma 2: Let R be semiprime ring and $F: R \to R$ be a generalized skew derivation associated with skew derivation d. If $F(uv) = 0$, for all $u, v \in R$, then $F = 0$ and $d = 0$.

Proof: We have $F(uv) = 0$, for all $u, v \in R$. (4) By replacing u by uw in equation (4), we get

$F(uwv) = 0$

$F(uw)v + \alpha(uw)d(v) = 0$

Using equation (4) in the above equation, we get

$\alpha(uw)d(v) = 0$, for all $u, v, w \in R$. (5)

Left multiplying equation (5) by $\alpha(w)d(v)$, we get.

$\alpha(w)d(v)\alpha(u)\alpha(w)d(v) = 0$, for all $u, v, w \in R$. Since α is an automorphism of R, we get.

$\alpha(w)d(v)R\alpha(w)d(v) = 0$, for all $v, w \in R$. Since R is semiprime ring, we get.

$\alpha(u)d(v) = 0$, for all $u, v \in R$. (6)

Left multiplying equation (6) by $d(v)$, we get.

$d(v)\alpha(u)d(v) = 0$, for all $u, v \in R$. Since α is an automorphism of R, we get.

$d(v)Rd(v) = 0$, for all $v \in R$. Since R is semiprime ring, we get.

$d(v) = 0$, for all $v \in R$. (7)

By the hypothesis $F(uv) = 0$, for all $u, v \in R$.

$F(u)v + \alpha(u)d(v) = 0$, for all $u, v \in R$.

Using equation (7) in the above equation, we get

$F(u)v = 0$, for all $x \in R$. (8)

Right multiplying equation (8) by $F(u)$, we get.

$F(u)vF(u) = 0$, for all $u, v \in R$, we get.

$F(u)RF(u) = 0$, for all $u \in R$. Since R is semiprime ring, we get.

$F(u) = 0$, for all $u \in R$.

Lemma 3: Let R be a semiprime ring and $F: R \to R$ be a generalized skew derivation associated with skew derivation d. If $F(uv) \in C_{1,\alpha}$, for all $u, v \in R$, then $[d(u), u]_{1,\alpha} = 0$, for all $u \in R$.

Proof:

We have $F(uv) \in C_{1,\alpha}$, for all $u, v \in R$. (9)

By replacing v by vw in equation (9), we get

$F(uvw) \in C_{1,\alpha}$

$F(uv)w + \alpha(uv)d(w) \in C_{1,\alpha}$, for all $u, v, w \in R$.

Using equation (9) in the above equation, we get

$\alpha(uv)d(w) \in C_{1,\alpha}$, for all $u, v, w \in R$.

$[\alpha(uv)d(w), w]_{1,\alpha} = 0$, for all $u, v, w \in R$.

$\alpha(u)\alpha(v)[d(w), w]_{1,\alpha} + [\alpha(u)\alpha(v), \alpha(w)]d(w) = 0$

$\alpha(u)\alpha(v)[d(w), w]_{1,\alpha} + \alpha(u)[\alpha(v), \alpha(w)]d(w) + [\alpha(u), \alpha(w)]\alpha(v)d(w) = 0$, for all $u, v, w \in R$.

By replacing v by w in the above equation, we get

$\alpha(u)\alpha(w)[d(w), w]_{1,\alpha} + [\alpha(u), \alpha(w)]\alpha(w)d(w) = 0$, for all $u, w \in R$.

By replacing w by u in the above equation, we get

$\alpha(u)\alpha(u)[d(u), u]_{1,\alpha} = 0$, for all $u \in R$. (10)

Left multiplying equation (10) by $\alpha(u)[d(u), u]_{1,\alpha}$, we get.

$\alpha(u)[d(u), u]_{1,\alpha}\alpha(u)\alpha(u)[d(u), u]_{1,\alpha} = 0$, for all $u \in R$. Since α is an automorphism, we get.

$\alpha(u)[d(u), u]_{1,\alpha}R\alpha(u)[d(u), u]_{1,\alpha} = 0$, for all $u \in R$. Since R is semiprime ring, we get.

$\alpha(u)[d(u), u]_{1,\alpha} = 0$, for all $u \in R$. (11)

Left multiplying equation (11) by $[d(u), u]_{1,\alpha}$, we get.

$[d(u),u]_{1,\alpha}\alpha(u)[d(u),u]_{1,\alpha}=0$, for all $u\in R$. Since α is an automorphism of R, we get.

$[d(u),u]_{1,\alpha}R[d(u),u]_{1,\alpha}=0$, for all $u\in R$. Since R is semiprime ring, we get.

$[d(u),u]_{1,\alpha}=0$, for all $u\in R$.

Lemma 4: Let R be a semiprime ring, $F:R\to R$ be a generalized skew derivation associated with skew derivation d and $H:R\to R$ be a left centralizer. If the map $G:R\to R$ is defined as $G(u)=F(u)\mp H(u)$, for all $u\in R$, then G is a generalized skew derivation associated with skew derivation d.

Proof:

Suppose that $G(u)=F(u)\mp H(u)$, for all $u\in R$. (12)

By replacing u by uv in equation (12), we get

$G(uv)=F(uv)\mp H(uv)$

$=F(u)v+\alpha(u)d(v)\mp H(u)v$

$=\big(F(u)\mp H(u)\big)v+\alpha(u)d(v)$, for all $u,v\in R$.

Using equation (12) in the above equation, we get

$G(uv)=G(u)v+\alpha(u)d(v)$, for all $u,v\in R$.

Then G is a generalized skew derivation associated with skew derivation d.

Theorem 1: Let R be a semiprime ring, $F:R\to R$ be a generalized skew derivation associated with skew derivation d and $H:R\to R$ be a left centralizer. If $F(uv)\mp H(uv)=0$, for all $u,v\in R$, then $d=0$. Moreover, $F(uv)=F(u)v$, for all $u,v\in R$ and $F=\pm H$.

Proof: By the hypothesis, we have $F(uv)-H(uv)=0$, for all $u,v\in R$.

Using equation (12) in the above equation, we get

So, we have $G(uv)=0$, for all $u,v\in R$.

Using lemma 2 and lemma 4, we get $G=0$.

So, we have $F=H$. (13)

By the hypothesis, we have $F(uv)-H(uv)=0$, for all $u,v\in R$.

$F(u)v+\alpha(u)d(v)-H(u)v=0$, for all $x\in R$.

Using equation (13) in the above equation, we get

$\alpha(u)d(v)=0$, for all $u,v\in R$. (14)

The equation (14) is same as equation (6) in lemma 2. Thus, by same argument of lemma 2, we can conclude the result $d(u)=0$, for all $u\in R$. (15)

By the definition of F, we have $F(uv)=F(u)v+\alpha(u)d(v)$, for all $u,v\in R$.

Using equation (15) in the above equation, we get

$F(uv)=F(u)v$, for all $u,v\in R$.

Similar proof shows that the same conclusion holds as $F(uv)+H(uv)=0$, for all $u,v\in R$. In this case, we obtain $F=-H$. Therefore, the proof is completed.

Theorem 2: Let R be a semiprime ring, $F:R\to R$ be a generalized skew derivation associated with skew derivation d and $H:R\to R$ be a left centralizer. If $F(uv)\mp H(vu)=0$, for all $u,v\in R$, then $d=0$. Moreover, $F(uv)=F(u)v$ for all $u,v\in R$ and $[F(u),u]=0$, for all $u\in R$.

Proof:

By the hypothesis, we have $F(uv) - H(vu) = 0$, for all $u, v \in R$. (16)

By replacing v by vw in equation (16), we get

$F(uvw) - H(vwu) = 0$

$F(uv)w + \alpha(uv)d(w) - H(vw)u = 0$, for all $u, v, w \in R$.

Using equation (16) in the above equation, we get

$H(vu)w + \alpha(uv)d(w) - H(vw)u = 0$

$\alpha(uv)d(w) + H(v)[u, w] = 0$, for all $u, v, w \in R$. (17)

By replacing w by u in equation (17), we get

$\alpha(uv)d(u) = 0$, for all $u, v \in R$. (18)

The equation (14) is same as equation (5) in lemma 2. Thus, by same argument of lemma 2, we can conclude the result $d(u) = 0$, for all $u \in R$. (19)

By the definition of F, we have $F(uv) = F(u)v + \alpha(u)d(v)$, for all $u, v \in R$.

Using equation (19) in the above equation, we get

$F(uv) = F(u)v$, for all $u, v \in R$. (20)

Using equation (19) in equation (17), we get

$H(v)[u, w] = 0$, for all $u, v, w \in R$. (21)

By replacing v by vw in equation (21), we get

$H(vw)[u, w] = 0$, for all $u, v, w \in R$.

Using equation (16) in the above equation, we get

$F(wv)[u, w] = 0$, for all $u, v, w \in R$.

Using equation (20) in the above equation, we get

$F(w)v[u, w] = 0$, for all $u, v, w \in R$. (22)

By replacing v by wv in equation (22), we get

$F(w)wv[u, w] = 0$, for all $u, v, w \in R$. (23)

Left multiplying equation (22) by w, we get.

$wF(w)v[u, w] = 0$, for all $u, v, w \in R$. (24)

By subtracting equation (24) from equation (23), we get

$[F(w), w]v[u, w] = 0$, for all $u, v, w \in R$.

By replacing u by $F(w)$ in the above equation, we get

$[F(w), w]v[F(w), w] = 0$, for all $v, w \in R$.

Since α is an automorphism of R, we get $[F(w), w]R[F(w), w] = 0$, for all $w \in R$.

Since R is semiprime ring, we get $[F(u), u] = 0$, for all $u \in R$.

Similar proof shows that the same conclusion holds as $F(uv) + H(vu) = 0$, for all $u, v \in R$. Therefore, the proof is completed.

Theorem 3: Let R be a semiprime ring, $F: R \to R$ be a generalized skew derivation associated with skew derivation d and $H: R \to R$ be a left centralizer. If $F(u)F(v) \mp H(uv) = 0$, for all $u, v \in R$, then $d = 0$. Moreover, $F(uv) = F(u)v$, for all $u, v \in R$ and $[F(u), u] = 0$, for all $u \in R$.

Proof:

By the hypothesis, we have $F(u)F(v) - H(uv) = 0$, for all $u, v \in R$. (25)

By replacing v by vw in equation (25), we get

$F(u)F(vw) - H(uvw) = 0$

$F(u)F(v)w + F(u)\alpha(v)d(w) - H(uv)w = 0$

$(F(u)F(v) - H(uv))w + F(u)\alpha(v)d(w) = 0$, for all $u, v, w \in R$.

Using equation (25) in the above equation, we get

$F(u)\alpha(v)d(w) = 0$, for all $u, v, w \in R$. (26)

By replacing u by tu in equation (26) and using equation (26), we get

$\alpha(t)d(u)\alpha(v)d(w) = 0$, for all $u, v, w, t \in R$.

By replacing v by vt and w by u in the above equation, we get

$\alpha(t)d(u)\alpha(v)\alpha(t)d(u) = 0$, for all $u, v, t \in R$. Since α is an automorphism of R, we get.

$\alpha(t)d(u)R\alpha(t)d(u) = 0$, for all $u, t \in R$. Since R is semiprime ring, we get.

$\alpha(t)d(u) = 0$, for all $u, t \in R$. (27)

The equation (27) is same as equation (6) in lemma 2. Thus, by same argument of lemma 2, we can conclude the result $d(u) = 0$, for all $u \in R$. (28)

Hence, from the definition of F, we get.

$F(uv) = F(u)v + \alpha(u)d(v)$, for all $u, v \in R$.

Using equation (28) in the above equation, we get

$F(uv) = F(u)v$, for all $u, v \in R$. (29)

By replacing u by uv in equation (25), we get

$F(uv)F(v) - H(uvv) = 0$, for all $u, v \in R$.

Using equation (29) in the above equation, we get

$F(u)vF(v) - H(uv)v = 0$, for all $u, v \in R$. (30)

Right multiplying equation (25) by v, we get.

$F(u)F(v)v - H(uv)v = 0$, for all $u, v \in R$. (31)

By subtracting equation (30) from equation (31), we get

$F(u)[F(v), v] = 0$, for all $u, v \in R$. (32)

By replacing u by ur in equation (32) and using equation (28), we get

$F(u)r[F(v), v] = 0$, for all $u, v, r \in R$. (33)

By replacing r by tr in equation (33), we get

$F(u)tr[F(v), v] = 0$, for all $u, v, r, t \in R$. (34)

Left multiplying equation (33) by t, we get.

$tF(u)r[F(v), v] = 0$, for all $u, v, r, t \in R$. (35)

By subtracting equation (35) from equation (34), we get

$[F(u), t]r[F(v), v] = 0$, for all $u, v, r, t \in R$.

By replacing t by u, v by u in the above equation and since α is an automorphism of R, we get $[F(u), u]R[F(u), u] = 0$, for all $u \in R$.

Since R is semiprime ring, we get $[F(u),u]=0$, for all $u\in R$.

Similar proof shows that the same conclusion holds as $F(u)F(v)+H(uv)=0$, for all $u,v\in R$. Therefore, the proof is completed.

Theorem 4: Let R be a semiprime ring, $F:R\to R$ be a generalized skew derivation associated with skew derivation d and $H:R\to R$ be a left centralizer. If $F(uv)\mp H(uv)\in C_{1,\alpha}$, for all $u,v\in R$, then $[d(u),u]_{1,\alpha}=0$, for all $u\in R$.

Proof:

By the hypothesis, we have $F(uv)\mp H(uv)\in C_{1,\alpha}$, for all $u,v\in R$.

Using equation (12) in the above equation, we get

Therefore, we have $G(uv)\in C_{1,\alpha}$, for all $u,v\in R$.

Using lemma 3 and lemma 4, we get $[d(u),u]_{1,\alpha}=0$, for all $u\in R$.

Theorem 5: Let R be a semiprime ring, $F:R\to R$ be a generalized skew derivation associated with skew derivation d and $H:R\to R$ be a left centralizer. If $F(uv)\mp H(vu)\in C_{1,\alpha}$, for all $u,v\in R$, then $[d(u),u]_{1,\alpha}=0$, for all $u\in R$.

Proof:

By the hypothesis, we have $F(uv)-H(vu)\in C_{1,\alpha}$, for all $u,v\in R$. (36)

By replacing v by vw in equation (36), we get

$F(uvw)-H(vwu)\in C_{1,\alpha}$

$F(uv)w+\alpha(uv)d(w)-H(vw)u\in C_{1,\alpha}$

$(F(uv)-H(vu))w+\alpha(uv)d(w)+H(vu)w-H(vw)u\in C_{1,\alpha}$

Using equation (36) in the above equation, we get

$\alpha(uv)d(w)+H(v)[u,w]\in C_{1,\alpha}$, for all $u,v,w\in R$.

$[\alpha(uv)d(w)+H(v)[u,w],w]_{1,\alpha}=0$

$[\alpha(uv)d(w),w]_{1,\alpha}+[H(v)[u,w],w]_{1,\alpha}=0$

$\alpha(uv)[d(w),w]_{1,\alpha}+[\alpha(uv),\alpha(w)]d(w)+[H(v)[u,w],w]_{1,\alpha}=0$

$\alpha(uv)[d(w),w]_{1,\alpha}+\alpha(u)[\alpha(v),\alpha(w)]d(w)+[\alpha(u),\alpha(w)]\alpha(v)d(w)+[H(v)[u,w],w]_{1,\alpha}=0$, for all $u,v,w\in R$.

By replacing w by u in the above equation, we get

$\alpha(uv)[d(u),u]_{1,\alpha}+\alpha(u)[\alpha(v),\alpha(u)]d(u)=0$, for all $u,v\in R$.

By replacing v by u in th above equation, we get

$\alpha(uu)[d(u),u]_{1,\alpha}=0$, for all $u\in R$. (37)

The equation (37) is same as equation (10) in lemma 3. Thus, by same argument of lemma 3, we can conclude the result $[d(u),u]_{1,\alpha}=0$, for all $u\in R$.

Similar proof shows that the same conclusion holds as $F(uv)+H(vu)\in C_{1,\alpha}$, for all $u,v\in R$. Therefore, the proof is completed.

Theorem 6: Let R be a semiprime ring, $F:R\to R$ be a generalized skew derivation associated with skew derivation d and $H:R\to R$ be a left centralizer. If $F(u)F(v)\mp H(uv)\in C_{1,\alpha}$, for all $u,v\in R$, then $[d(u),u]_{1,\alpha}=0$, for all $u\in R$.

Proof:

By the hypothesis, we have $F(u)F(v) - H(uv) \in C_{1,\alpha}$, for all $u, v \in R$. (38)

By replacing v by vw in the equation (38), we get

$F(u)F(vw) - H(uvw) \in C_{1,\alpha}$

$F(u)F(v)w + F(u)\alpha(v)d(w) - H(uv)w \in C_{1,\alpha}$

$(F(u)F(v) - H(uv))w + F(u)\alpha(v)d(w) \in C_{1,\alpha}$, for all $u, v, w \in R$.

Using equation (38) in the above equation, we get

$F(u)\alpha(v)d(w) \in C_{1,\alpha}$, for all $u, v, w \in R$. (39)

By replacing u by tu in equation (39) and using equation (39), we get

$\alpha(t)d(u)\alpha(v)d(w) \in C_{1,\alpha}$

$[\alpha(t)d(u)\alpha(v)d(w), w]_{1,\alpha} = 0$, for all $u, v, w, t \in R$.

By replacing $\alpha(t)d(u)$ by $\alpha(u)$ in the above equation, we get

$[\alpha(uv)d(w), w]_{1,\alpha} = 0$, for all $u, v, w \in R$.

$\alpha(uv)[d(w), w]_{1,\alpha} + [\alpha(uv), \alpha(w)] = 0$

$\alpha(uv)[d(w), w]_{1,\alpha} + \alpha(u)[\alpha(v), \alpha(w)] + [\alpha(u), \alpha(w)]\alpha(v) = 0$

By replacing w by u in th above equation, we get

$\alpha(uv)[d(u), u]_{1,\alpha} + \alpha(u)[\alpha(v), \alpha(u)] = 0$, for all $u, v \in R$.

By replacing v by u in th above equation, we get

$\alpha(uu)[d(u), u]_{1,\alpha} = 0$, for all $u \in R$. (40)

The equation (40) is same as equation (10) in lemma 3. Thus, by same argument of lemma 3, we can conclude the result $[d(u), u]_{1,\alpha} = 0$, for all $u \in R$.

Similar proof shows that the same conclusion holds as $F(u)F(v) + H(uv) \in C_{1,\alpha}$, for all $u, v \in R$. Therefore, the proof is completed.

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