IJRAR.ORG E-ISSN: 2348-1269, P-ISSN: 2349-5138

INTERNATIONAL JOURNAL OF RESEARCH AND ANALYTICAL REVIEWS (IJRAR) | IJRAR.ORG

An International Open Access, Peer-reviewed, Refereed Journal

Equality of G-Radical and G^1 Radical for an Ideal in a Non-Associative Ring

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Abstract : In this study, we described the statement "Equality of G-Radical and G^1 Radical for an Ideal in a Non-Associative Ring" addresses an interesting property in non-associative ring theory. In this context, it asserts that for any ideal D within a non-associative ring R, the G-radical and G^1 radical of D are identical. This implies that the elements that are "nilpotent" with respect to right multiplication (G-radical) are also "nilpotent" with respect to left multiplication (G^1 radical) within the specified ideal D. This abstract summarizes the core concept without delving into the specific mathematical details, highlighting the relationship between these two radicals in non-associative rings.

IndexTerms - Prime ideal, non-associative ring , G – radical, nilpotent ideals.

1. Introduction

1. Radicals G and G^1:

In nonbonding rings, the term radical is used to define a subset or ideal of "zero" or "zero". In this context, radical G and radical G^1 are two different concepts of radical [1].

Radical G (G(D)): A radical of an ideal D in a non-binding cycle R of G is defined as the intersection of all right maximal ideals containing D. That is, it consists of all elements x of R. natural number n, for which x^n is D.

Radical G^1 (G^1(D)): A radical of an ideal D in a nonbonding ring R of G^1 is defined as the intersection of all left-maximal ideals containing D. Like the radical G it consists of all elements. x in R. This element has the natural number n, so x^n in D [2-4].

2. Detailed examination of the declaration:

The statement you make is: "Assuming that D is an unbonded ring ideal of UR, the radical G and the radical G^1 in D are the same."

To understand and analyze this statement, let's look at some key points.

This statement assumes that R is a disjoint circle. This means that multiplication operations are not required in R to satisfy the associative property. This deviation from associativity can make the study of radicals more difficult than that of associative rings[5-6].

D is considered the ideal of R. Ideally, a subset of the cycle is closed during aggregation and absorbs the products of the cycle. In this context, D is the ideal of R. This statement says that the radicals G and G^1 in D are equal. This means that an element that is “nullpotent” in right-hand multiplication (radical G) is also “nullpotent” in left-hand multiplication (radical G^1).

Reasons for this statement:
The equivalence of the radicals G and G^1 with respect to the ideal D of the non-associative circle R can be preserved by certain properties and properties of the non-associative circle. The details and proof of such a statement require a detailed analysis of the unbonded ring theory and the properties of the G and G^1 radicals [7].
To further investigate this statement and obtain a precise proof, we need to take a closer look at the specific properties and axioms of the non-associative ring, its ideals and the behavior of its elements in relation to right and left multiplication. This analysis generally requires mathematical precision and may depend on the specific definition and properties of the unbonded rings in question [8].

In short, the statements you made suggest interesting properties of G and G^1 radicals in unbound cycles, but detailed proof or study would require a deeper study of unbound cycle theory and related concepts.

2. LEMMA

LEMMA : Suppose P is an ideal in the U-non-Associative ring R. Then the following are true.
(i) If$D1$,$D2$.....Du are ideals in R and $D1$,$D2$ $Du \subseteq P thek\ D1 \subseteq P,(or) D2 \subseteq P$..(or)$Du \subseteq P$
(ii) If$D1$,$D2$.....Du are ideals in R and $D1 \cap \bar{P} \neq \emptyset$, $D2 \cap \bar{P} \neq \emptyset$, ... and $Du \cap \bar{P}$ then $D1$,$D2$ $Du \cap \bar{P} \neq \emptyset$ [$\bar{P}$ *is the complet* set of P]
(iii) If $t1, t2$.....tu are elements in $\bar{P} thek$ $[t1], [t2]$ $[tu] \cap \bar{P}$G$\emptyset$ where $[ti]$ is the ideal generated by ti

Proof: (i) $\Rightarrow (ii)$ *akd also* $(iii) \Rightarrow (ii)$
To prove that $(ii) \Rightarrow (iii)$
Let$D1$, $D2$ Du are ideals in R with $D1 \cap \bar{P}$G$\emptyset$,
$D2 \cap P$G$\emptyset$,......$Du \cap$ PG$\emptyset$.
Then the rear elements
$t1ikD1 \cap \bar{P}$, $t2ikD2 \cap \bar{P}$,......$and\ tu\ in\ Du \cap \bar{P}$.
But by(iii), $[t1][t1]$......$[tu]$G$\emptyset$.
So we have $D1$,$D2$.....$Du \cap \bar{P} \geqslant [t1][t1]$.....$[tu] \cap \bar{P}G\emptyset$,
and hence $D1$,$D2$.....$Du \cap \bar{P}$G$\emptyset$.

3. THEOREM

THEOREM : Suppose D is ideal in the U-non associative ring R then D^G is the intersection of all the prime ideals P which contain D.
The proof involves two parts:
Forward Direction: If an element h belongs to D_G, then it is shown that h also belongs to every prime ideal P containing D. This is established by invoking Lemma (1) to identify a G-system (denoted as P̅) that does not intersect D. Then, by the definition of D_G, it is concluded that h must belong to P. Thus, D_G is contained in every prime ideal containing D.
Converse Direction: If an element h does not belong to D_G, it is shown that there exists a maximal ideal P containing D such that the intersection of P and a G-system W, denoted as P ∩ W, is empty. It is also proved that P is a prime ideal in R. This part uses the fact that P is a maximal ideal containing D that does not intersect W.
Proof: Suppose $h \in D^G$, and P is the prime ideal containing D. from Lemma (1) we say that $\bar{P}$ is a G- system not meeting
D, So that by the definition of D^G we have $h \in P$.Thus
D^G is contained in every prime ideal P containing D and hence is contained in their intersection.
In the converse case, suppose $h\emptyset\ D^G$. Then there is a G- system W containing h such that W $\cap D = \emptyset$. by applying Zorn's lemma [6] to get a maximal ideal P containing D such that $P \cap$ W=$\emptyset$. Now we prove that P is a prime ideal in
R. Let $D1$,$D2$,.....Du are ideal in R such that Dj$\cap \bar{P}$G
$\emptyset\forall$ since P is a maximum ideal containing D which does not meet W, the ideals P+$D1$, P+$D2$, P+Du, be

the ideals containing D, all meet W. Hence $(D1+P)$, $(D2+P)$, ...$(Du+ P)$ meets W But if $(t1+P1)(t2+P2).....(tn+Pn)$ is an element of $(D1+P)(D2 + P)$ .....$(Du + P)$, where the elements are multiplied then $(t1+P1)(t2+ P2).....(tn+Pn) = t1t2....tn+$ the sum of terms of the form $t1t2....Pn$ ...$Pn...tn$. But each of the terms except the first is clearly in P.

Therefore

$(D_1+P)(D_2+ P)...(Dn+ P)\subseteq D_1D_2.....D_n+ P$.

Therefore P is Prime and $h\o P$

The presented theorem and its proof demonstrate a fundamental relationship between the G-radical of an ideal D and prime ideals containing D in a non-associative ring. The results can be summarized as follows:
D_G, the G-radical of D, is the intersection of all prime ideals containing D.
The forward direction shows that if an element is in D_G, it must belong to all prime ideals containing D.
The converse direction shows that if an element is not in D_G, there exists a maximal ideal P containing D that is a prime ideal in R.

4. Conclusion

In conclusion, this theorem and its proof provide insights into the structure of prime ideals in non-associative rings and their relationship with the G-radical of ideals. It helps in understanding how elements behave in relation to these prime ideals and the G-radical. These results are important for studying the properties and characteristics of non-associative rings and their ideals.

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